

# Vertical Fiscal Externalities and Federal Tax-Transfers under Variable Factor Supplies

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## Abstract

Within a model of variable supply of capital due to international mobility and variable labor supply due to endogenous labor-leisure choice, we revisit the issues of vertical fiscal externalities, and of federal tax-transfers. Capital and labor taxes by federal and state governments finance the provision of federal and of state public consumption goods. When capital and labor are substitutes in production, we show that (i) the state's optimal policy calls for capital and labor taxes, (ii) the vertical fiscal externality can be reversed from negative, implying inefficiently high noncooperative capital taxes, to positive, implying inefficiently low noncooperative capital taxes, and (iii) under centralized leadership the federal government replicates the second best optimum with a capital tax, and possibly, top-down transfers. (JEL codes: F18, F21, H21).

**Key words:** fiscal federalism, vertical fiscal externalities, bottom-up and top-down transfers, variable factor supplies

## 1. Introduction

More often than not, in a federal economic system, for example, the USA, the EU or Canada, decentralized, that is, regional or national, fiscal policies may influence the fiscal actions of other regional/national, or even federal authorities. For example, a prevalent feature in federal economies is the co-sharing or the co-occupancy of the same tax-bases between the different levels of fiscal authority, for example, between different tiers of

government, such as federal (high-level) and state (low-level).<sup>1</sup> This co-sharing of tax-bases potentially gives rise to a so-called ‘common pool problem’ whereby tax decisions by one level of government affect tax-payers decisions, which in turn affect the shared tax-bases. In this way, changes in state taxes can entail non-negligible effects on federal budgets and vice-versa. In the core of this common pool problem two issues of importance are the so-called ‘vertical fiscal externalities’ and ‘bottom-up’ or ‘top-down fiscal transfers’, or else negative or positive *fiscal gap*. Understanding the interaction of tax revenues and tax rates between federal and state governments is a significant issue in the design and implementation of federal tax systems.

‘Vertical fiscal externalities’ arise when state governments levy taxes discounting the effect on federal tax revenues and consequently the effect on other states in so far that the latter are entitled to a share of the federal tax revenues.<sup>2</sup> A pivotal conclusion of the relevant literature which considers models with a single factor in variable supply is that when tax-bases are shared among different tiers of government, and tax rates are specific, the emerging ‘vertical fiscal externalities’ are negative, implying inefficiently high non-cooperative specific taxes (e.g., [Boadway and Keen 1996](#); [Boadway et al. 1998](#); [Keen 1998](#); [Hoyt 2001](#); [Keen and Kotsogiannis 2004](#); [Köthenbürger 2004](#), [Boadway and Tremblay 2012](#)).<sup>3</sup> With ad valorem taxes, however, the ‘vertical fiscal externality’ can be either positive or negative (e.g., [Dahlby and Wilson 2003](#); [Kotsogiannis and Martinez 2008](#); [Karakosta 2010](#); [Sas 2017](#)).<sup>4</sup>

‘Bottom-up’ and ‘top-down transfers’ are the resource transfers from low to high-level governments, and vice-versa. The main conclusions of the relevant literature are that when

- 1 There is ample real-world evidence of tax basis co-occupation by local and central governments in federal states. For example, the USA taxes are levied on labor and corporate incomes, capital gains, dividends, etc., by the federal and state governments. In Canada, both the federal and provincial governments impose taxes on labor income and dividends. In Switzerland, federal and cantonal taxes are imposed on wages, salaries, rent, interest payments, and profits for an overview of co-occupied tax bases in federal and unitary states (see among others [Boadway and Shah 2009](#); [Bizioli and Sacchetto 2011](#)).
- 2 Vertical fiscal externalities differ from horizontal fiscal externalities, the latter is due to the tax competition among state governments which results to horizontal mobility of tax-bases among them (e.g., [Bucovetsky and Wilson 1991](#); [Wilson and Wildasin 2004](#); [Ogawa et al. 2016](#)). While horizontal fiscal externalities are likely to lead to low state taxes, the vertical fiscal externalities usually lead to high state taxes.
- 3 For a survey on vertical fiscal externalities see, among others, [Keen \(1998\)](#).
- 4 An extensive empirical literature attests to the prevalence of vertical fiscal externalities in federal fiscal systems. [Hayashi and Boadway \(2001\)](#) provide evidence for the existence of both vertical and horizontal externalities in the case of the Canadian business income taxes. [Esteller-Moré and Solé-Ollé \(2002\)](#) estimate vertical income tax externalities in the setting of income taxes in Canada. [Brülhart and Jametti \(2006\)](#) show that vertical externalities dominate horizontal ones in the setting of Swiss municipalities local taxes. [Devereux et al. \(2007\)](#) study USA state and federal excise taxes on cigarettes and gasoline. They show that in the case of cigarette taxes the tax set in any state responds positively to taxes set in neighboring states, but unresponsive to the federal tax. In the case of gasoline taxes the tax set in any state is unresponsive to taxes set in neighboring states, but responsive to the federal tax and this response can be of either sign. [Reingewertz \(2018\)](#) provides evidence for the influence of federal tax shocks on state tax revenues. [Boadway and Tremblay \(2012\)](#) provide a general review of the theoretical and empirical literature on fiscal federalism.

the federal government pre-commits vis-a-vis the state governments, that is, centralized leadership, and tax bases are shared between them, then the federal government replicates, at all times, the unitary government's optimal (second-best) tax setting in the presence of transfers (e.g., [Boadway and Keen 1996](#); [Boadway and Tremblay 2006](#); [Kotsogiannis and Martinez 2008](#); [Sas 2017](#)).<sup>5</sup> If, however, federal transfers are missing, then the federal government cannot replicate the unitary government's second-best policy outcome (e.g., [Keen and Kotsogiannis 2002](#)).<sup>6</sup> When tax bases do not perfectly overlap, then under centralized leadership the federal government cannot replicate the unitary government's second-best policy, even in the presence of federal transfers (e.g., [Koethenbueger 2008](#)).<sup>7</sup>

The objective of the paper is to re-examine the issues of the 'vertical fiscal externality' and of the 'bottom-up/top-down transfers', and their relationship in a model of a multistates, or jurisdictions, federal economy with federal-cum-state specific factor taxes, and where capital and labor are in variable supplies. Variable supply of capital is due to international mobility, across the states of the federation as well as between the federation and the rest of the world. The supply of labor is variable due to endogenous labor-leisure decision. Specific capital and labor taxes by the federal government finance the provision of a federal public consumption good, and specific capital and labor taxes by a state government finance the provision of a state public consumption good. Within this framework, we first derive the state's simultaneous optimal setting of capital and labor taxes. We show that when labor and capital are substitutes in production, the optimal policy for a representative state calls for a capital and a labor tax. Then, we examine the impact of the capital and labor tax setting behavior by a state on the provision of the federal public consumption good, that is, 'the vertical fiscal externality'. When capital and labor are substitutes in production, the sign of the 'vertical fiscal externality' can be reversed from negative to positive, implying inefficiently low instead of high noncooperative capital taxes. Intuitively, when state governments raise their own specific capital tax rates, capital leaves the federation and, on the one hand the capital tax base of the federal government contracts and as a result federal tax revenues fall. According to the relevant literature this result leads to inefficiently high noncooperative specific taxes. On the other hand, a rise in state capital taxes increases labor supply when capital and labor are substitutes in production. This expands the labor tax base of the federal government, and the total federal tax revenues can increase. Thus, when capital and labor are in variable supply and taxes are specific, the 'vertical fiscal externality', under plausible conditions, can be reversed from negative to positive.

We examine a unitary government's optimal rule, that is, second-best optimum, for the provision of the local and federal public consumption goods, and we show that the federal

- 5 Precommitment, or so-called central leadership, by the federal government refers to the case where it acts as a Stackelberg leader.
- 6 [Haufler and Lülfesmann \(2015\)](#) show, in a sequential game, that the optimal choice of the federal tax rate in the first stage mitigates externalities that local taxes create. [Hoyt \(2001\)](#) examines the optimal federal tax and transfer policy when higher and lower tier governments share tax bases, but neither level of government can precommit when setting policy instruments (simultaneously determined policies).
- 7 [Köthenbürger \(2007\)](#) within a context where the federal government does not have access to a tax policy instrument, examines the optimal transfer policy with or without pre-commitment by the federal government. [Dréze et al. \(2007\)](#) examine, within a federation, the efficiency of voluntary matching grants schemes in terms of redistribution under perfect labor mobility.

tax-transfer policy can achieve this second-best optimum under centralized leadership. The optimal federal tax-transfer policy when capital and labor are substitutes in production requires a capital tax and, possibly, a ‘top-down transfer’. When capital and labor are complements in production the optimal federal policy is a capital subsidy, while transfers are ‘bottom-up’. Consequently, the relationship in production between capital and labor in variable supply is the driving force of our results.

## 2. The Model

Consider a federal economy with  $N$  symmetric states, populated with  $\Lambda \geq 1$  identical and immobile across states individuals. Since households are assumed identical, following standard practice, we normalize  $\Lambda = 1$ , thus  $N$  also denotes the overall population of the federal economy. The federal economy is assumed small in world commodity and capital markets. Thus, in the federation, all commodity and factor markets are perfectly competitive, and commodity prices of the internationally traded goods and the rate of return to the internationally mobile capital are fixed and exogenous.

In a representative state,  $M$  freely tradable consumption goods are produced, and production technologies are assumed identical across all states. Several primary factors are used in the production of the traded goods, among which, capital ( $K$ ), and labor ( $L$ ), are in variable supply. The supply of capital is variable due to international mobility, while the supply of labor, an internationally and interstate immobile factor, is variable due to endogenous labor-leisure choice. All other factors are immobile and in fixed endowments.

The production side is represented by the Gross Domestic Product (GDP) function. The GDP function is given by  $R(p, K, L, \Omega) = \max \left\{ \sum_{j=1}^M p_j x_j : F(X, K, L, \Omega) \leq 0 \right\}$  and denotes the maximum value of output production at world commodity prices  $p$ , given the country’s variable domestic supply of capital  $K$  and level of employment  $L$ , and the vector  $\Omega$  of all other internationally immobile and in fixed supplies factors.  $F(X, K, L, \Omega)$  is the aggregate production possibilities set, where  $X$  is the vector of goods produced. For simplicity, the vectors  $(p, \Omega)$  are omitted from the subsequent analysis because of our assumptions regarding fixed world commodity prices and fixed supplies of all other factors. Then, the GDP function reduces to  $R(K, L)$ .<sup>8</sup>  $K = \bar{K} + k$  is the supply of capital in the representative state,  $\bar{K}$  is the state’s own capital endowment. If  $k < 0$ , the state is a net capital exporter to all other states of the federation and to the rest of the world, and if  $k > 0$ , the state is a net capital importer. The derivatives of the  $R(\cdot)$  function with respect to  $K$  and  $L$ , that is,  $R_K = \partial R / \partial K$  and  $R_L = \partial R / \partial L$ , denote the marginal revenue products of capital and labor. Strict concavity is assumed of the  $R(\cdot)$  function with respect to  $K$  and  $L$ , that is,  $R_{KK} (= \partial R_K / \partial K) < 0$  and  $R_{LL} (= \partial R_L / \partial L) < 0$ .<sup>9</sup> Furthermore,  $R_{KL} (= \partial R_K / \partial L) = R_{LK}$ , can be

8 The GDP function is a maximum value function which captures the entire production behavior of an economy, for example, production technologies, profit conditions, resource constraints, etc. (see Raimondos-Møller and Woodland 2006; Anderson and Neary 2016).

9 Subscripts denote partial derivatives.

either positive or negative, implying respectively, that capital and labor can be either complements or substitutes in production.<sup>10</sup>

A representative household derives utility from the consumption of the traded goods, leisure ( $l$ ), and the two public consumption goods, all of which are considered normal in consumption.<sup>11</sup> A ‘state’ public consumption good ( $g$ ) provided free of charge by the state authority to its residents, and a ‘federal’ public consumption good ( $G$ ) provided free of charge by the federal authority to the residents of all states in the federation. Both these goods are internationally traded and their prices are given. The representative household’s preferences are depicted by the minimum expenditure function  $E(g, G, L, u) = \min \left\{ \sum_{j=1}^{j=M} p_j C_j \mid U(C_1, \dots, C_M, 1 - L, g, G) \geq u \right\}$ .<sup>12</sup> It denotes the minimum expenditure on the traded goods required to attain a given level of utility ( $u$ ), given the levels of leisure ( $l = 1 - L$ ), and of consumption of the state and federal public goods,  $g$  and  $G$ . The fixed commodity prices are tacitly omitted as arguments from the  $E(\cdot)$  function. The derivatives of the  $E(\cdot)$  function,  $E_g$  and  $E_G$  are negative implying that an increase in the consumption of the state and federal public consumption goods reduces the expenditure on private goods required to achieve a given level of utility  $u$  (e.g., [Kotsogiannis et al. 2005](#); [Raimondos-Møller and Woodland 2006](#)). Thus,  $-E_g > 0$  and  $-E_G > 0$  denote respectively, the so-called marginal willingness to pay for the consumption of the state and federal public goods, and  $-E_{gg} < 0$  and  $-E_{GG} < 0$ . The derivative  $E_u > 0$ , is the inverse of the marginal utility of income, and  $E_L$  captures the representative household’s reservation wage, denoted by  $\tilde{w}$ , that is,  $E_L = \tilde{w}$  (e.g., [Anderson and Neary 2016](#)).<sup>13</sup> The  $E(\cdot)$  function is strictly convex in  $L$ , that is,  $E_{LL} > 0$ , implying that an increase in labor supply, reduces leisure and thus it increases the reservation wage.

- 10 From a theoretical viewpoint, in a two factors,  $K$  and  $L$ , model, the two factors must be substitutes in production (e.g., [Borjas 2016](#)). In models with more than two factors,  $K$  and  $L$  can be substitutes, complements, or independent in production (e.g., [Koethenburger 2008](#)). In models with many inputs and different types of workers, for example, skilled and unskilled labor, empirical evidence suggests that capital and skilled labor are complements, while unskilled labor and capital are substitutes (e.g., [Goldin and Katz 1998](#); [Borjas 2016](#)). Capital and labor are substitutes in sectors such as footwear industries. For example, in developing economies footwear is produced using a high labor-capital ratio, while the same products in developed economies are produced with a low labor-capital ratio. This is explained by the fact that the cost of labor is low in developing economies and high in developed ones (e.g., [Feenstra 2016](#), p.9).
- 11 Let  $l$  and  $L$  respectively denote leisure and labor supply. Then, we can normalize the time endowment to 1, so that  $l + L = 1$ .
- 12 A utility function compatible with these assumptions is an additively separable function, for example,  $U(C_1, \dots, C_M, 1 - L, g, G) = \nu(C_1, \dots, C_M, 1 - L) + \psi(g) + \Psi(G)$ , where the subutility  $\nu(C_1, \dots, C_M, 1 - L)$  is quasi-linear and increasing in consumptions and leisure, with income effects falling on the numeraire commodity, say good 1.  $U$  is increasing and concave in  $g$  and  $G$  (e.g., see [Aronsson et al. 2014](#)).
- 13 This modeling is quite prevalent in international trade literature that examines various issues, such as welfare effects from trade liberalization under endogenous labor supply within a perfectly competitive context (e.g., [Mayer 1991](#); [Li and Mayer 1990](#); [Woodland 1982](#)). Recently, [Arkolakis and Esposito \(2015\)](#) consider variable labor supply under monopolistic competition to re-examine the gains from international trade.

Employment is endogenous and workers supply labor until the endogenously determined reservation wage, that is,  $E_L(\cdot)$ , equals the net wage received.<sup>14</sup> The latter is defined as the marginal revenue product of labor, that is,  $R_L(\cdot)$ , minus the wage taxes paid to the state and federal governments. Thus, the representative state's labor market equilibrium condition can be written as:

$$E_L(g, G, L, u) = R_L(K, L) - \sigma, \tag{1}$$

where  $\sigma (= s + S)$ , is a consolidated labor tax,  $s$  is the state specific labor tax, and  $S$  is the corresponding federal one.

Because of the assumed international mobility of capital, equilibrium in the state capital market requires that the net rate of return to capital, that is, the marginal revenue product of capital minus the consolidated capital tax  $\tau (= t + T)$ , is equal to its world rate of return ( $\rho$ ).<sup>15</sup>  $t$  is the state and  $T$  is the federal specific capital tax.<sup>16</sup> That is:

$$R_K(K, L) - \tau = \rho. \tag{2}$$

The state and federal governments finance the provision of the public consumption goods,  $g$  and  $G$ , respectively. State tax rates ( $t, s$ ) are the strategic policy instruments of the state government, while federal taxes ( $T, S$ ) and resource (income) transfers  $F$  are the strategic policy instruments of the federal government. For simplicity, we assume that there are no horizontal, that is, among states, resource transfers, that the prices of  $g$  and  $G$  is one, and that both levels of government maintain a balanced budget. The state and the federal budget constraints are, respectively, given by:

$$g = sL + tK + F, \text{ and} \tag{3}$$

$$G = N(SL + TK - F), \tag{4}$$

where  $F$ , the 'vertical' lump-sum transfer between the federal and the state governments, can be either positive ( $F > 0$ ) or negative ( $F < 0$ ). Totally differentiating the budget constraints (3) and (4) we obtain the following comparative statics results:

- 14 For analytical tractability of the results we assume that the reservation wage is not affected by changes in public consumption goods and income, that is,  $E_{Lg} = E_{LG} = E_{Lu} = 0$ , (e.g., see [Keen and Kotsogiannis 2002](#); [Kotsogiannis and Martinez 2008](#)). Changes in  $E_L (= \bar{w})$  due to changes in  $\tau, \sigma, \tau^*, \sigma^*$  are channeled via changes in labor supply, that is,  $\frac{dE_L}{dj} = E_{LL} \left( \frac{dL}{dj} \right) = \frac{\partial \bar{w}}{\partial L} \left( \frac{dL}{dj} \right)$ ,  $j = \tau, \sigma, \tau^*, \sigma^*$ .
- 15 As previously noted, capital is mobile across states of the federation, and between the federation and the rest of the world. Since the federation is small in the world capital markets, the latter capital flight is more prevalent to the former, and thus higher capital taxes within the federation contract its capital tax base and result to lower capital tax revenue for the federal government. In related studies which consider only inter-state mobility of capital within a federal economy, for example, [Hoyt and Jensen \(1996\)](#) and [Koethenbueger \(2008\)](#), overall supply of capital in the federation remains fixed, due to the absence of international mobility.
- 16 In the present framework, and by and large in the relevant literature, state taxes are not deducted or credited against federal taxes. For this issue (see e.g., [Dahlby et al. 2000](#)).

$$\begin{aligned}
g_t &= \frac{dg}{dt} = K + \alpha, \quad g_T = \frac{dg}{dT} = \alpha, \quad g_F = 1, \quad \frac{dg}{dt^*} = \frac{dg^*}{dt} = 0, \\
G_t &= A, \quad G_T = N(K+A), \quad G_F = -N, \quad \text{and} \\
g_t &= g_T + K, \quad \text{and} \quad G_T = N(K+G_t),
\end{aligned} \tag{5}$$

where  $\alpha = \Delta^{-1}(sR_{LK} + tZ_{LL})$ ,  $A = \Delta^{-1}(SR_{LK} + TZ_{LL})$ , and by the properties of the GDP and minimum expenditure functions,  $\Delta = R_{KK}(Z_{LL} + R_{LK}R_{KK}^{-1}R_{KL}) < 0$  and  $Z_{LL} = E_{LL} - R_{LL} > 0$ . Equations (A.1), (A.2) and (A.3) in the Appendix provide detailed derivations and the analytical expressions of these results. We adopt the plausible assumptions that  $g_t > 0$  and  $G_T > 0$ , that is, a higher state/federal capital tax increases the provision of the state/federal public consumption good. This assumption requires  $(K + \alpha) > 0$  and  $(K + A) > 0$ .<sup>17</sup> The sign of  $G_t$  is ambiguous depending on whether  $R_{KL} > 0 (< 0)$ , that is, on whether  $K$  and  $L$  are complement (substitute) factors in production.<sup>18</sup> Specifically,  $G_t < 0$  if  $R_{KL} > 0$ , and  $G_t \leq 0$  if  $R_{KL} < 0$ . The result  $\frac{dg}{dt^*} = \frac{dg^*}{dt} = 0$  indicates that the capital tax policy by one state does not affect the provision of the state public good in the other, that is, it does not entail a horizontal fiscal externality.

To close the model, the representative state's income-expenditure identity requires that private spending on traded goods equals income from production minus capital and labor taxes and payments to capital from other states and the rest of the world used domestically, or plus the capital receipts for its capital employed abroad. That is:

$$E(g, G, L, u) = R(K, L) - \tau K - \sigma L - \rho k. \tag{6}$$

### 3. Optimal State Capital and Labor Taxes

First, we examine a representative state's simultaneous setting of capital and labor taxes that maximize its own welfare, assuming a passive Federal government in terms of its tax setting behavior. Differentiating equations (3), (4), and (6) with respect to  $(t, s)$ , assuming  $dS = dT = dF = 0$ , and using equations (A.2), we obtain:

$$\begin{aligned}
E_u \frac{du}{dt} &= -(E_g s + E_G S) \Delta^{-1} R_{LK} - (E_g t + E_G T) \Delta^{-1} Z_{LL} - (E_g + 1)K, \quad \text{and} \\
E_u \frac{du}{ds} &= (E_g s + E_G S) \Delta^{-1} R_{KK} - (E_g t + E_G T) \Delta^{-1} ZR_{KL} - (E_g + 1)L.
\end{aligned} \tag{7}$$

Setting  $E_u \frac{du}{dt} = E_u \frac{du}{ds} = 0$  and solving simultaneously, the optimally set capital and labor taxes are given by:

$$\begin{aligned}
t^{\text{opt}} &= -(E_g + 1)E_g^{-1}(KR_{KK} + LR_{LK}) - \epsilon T, \quad \text{and} \\
s^{\text{opt}} &= (E_g + 1)E_g^{-1}(LZ_{LL} - KR_{LK}) - \epsilon S,
\end{aligned} \tag{8}$$

where  $\epsilon = \frac{E_G}{E_g} > 0$ . Assuming that  $|E_g| > 1$ , implying that the marginal willingness to pay for the provision of the state public good is higher than its unit cost, that is, the public

17 This assumption implies that we are on the left side of a Laffer curve (see e.g., [Ogawa et al. 2006](#)).

18 Differentiating the equilibrium condition (2) we obtain  $\frac{dK}{dL} = -R_{KK}^{-1}R_{KL}$ . Then,  $\frac{dK}{dL} > 0$  if  $R_{KL} > 0$ , implying complementarity of the two factors in production, while  $\frac{dK}{dL} < 0$  if  $R_{KL} < 0$ , implying substitutability.

consumption good is under-provided.<sup>19</sup> Equations (8) indicate that when capital and labor are substitutes in production, that is,  $R_{LK} < 0$ , then (i) in the absence of a federal government, that is,  $T = S = F = 0$  (and so  $G = 0$ ), the representative state's optimal policy is jointly a capital and a labor tax,<sup>20</sup> and (ii) the presence of federal government, that is,  $T > 0, S > 0$  (and so  $G > 0$ ), but passive in terms of its tax/transfers setting behavior, that is,  $dS = dT = dF = 0$ , lowers the jointly optimal state taxes. Based on the above, we state the following Proposition.

**Proposition 1** *Consider a representative state in a federal economy, where capital and labor are in variable supplies. If capital and labor are substitutes in production, then the state's optimal policy calls for simultaneously positive capital and labor taxes. The existence of federal capital and labor taxes lowers the state's optimal tax rates.*

#### 4. Benchmark Case: The Unitary Government

Next, we consider the equilibrium outcome pursued by a single, that is, 'unitary', government, whose objective is to maximize the federal welfare  $W = Nu$  by choosing  $\tau$ , subject to the consolidated government budget constraint:

$$Ng + G = N(\sigma L + \tau K). \tag{9}$$

This analysis serves as a benchmark case to the alternative equilibrium setting of capital taxes by both levels of government, to be examined later on.<sup>21</sup> Differentiating the federal welfare function  $W = Nu$  with respect to the capital tax  $\tau$ , treating labor taxes exogenous, and using equations (1), (2), and (6) we obtain:<sup>22</sup>

$$\frac{1}{N}dW = E_u \frac{du}{d\tau} = -E_g \frac{dg}{d\tau} - E_G \frac{dG}{d\tau} - K. \tag{10}$$

Furthermore, from equation (9) we have:

$$N \frac{dg}{d\tau} + \frac{dG}{d\tau} = N \left( \sigma \frac{dL}{d\tau} + \tau \frac{dK}{d\tau} + K \right). \tag{11}$$

Combining equations (10) and (11), using equations (A.2), and assuming that  $dG = 0$ , the first-order condition for the optimal provision of  $g$  requires that the optimal capital tax is chosen so that:

- 19 In the presence of lump-sum taxes, the optimal provision of the public consumption good requires  $-E_g = 1$ .
- 20 In the case where capital and labor are complements in production, the sign of the optimal rates in equations (8) is ambiguous. However, in order to ensure positive tax revenue to finance the provision of the public consumption good, at least one of the two tax rates must be positive.
- 21 Now  $\sigma$  and  $\tau$  represent a single labor and capital income tax, respectively, imposed by the unitary government.
- 22 The assumption of holding a second tax policy instrument exogenous is a standard practice in the relevant literature (e.g., Keen and Kotsogiannis 2002; Dahlby and Wilson 2003).



$$\frac{1}{N}dW = E_u \frac{du}{d\tau} \Big|_{dG=0} = -E_g g_\tau - K = -E_g(\Phi + K) - K = 0, \quad (12)$$

where  $\Phi = \Delta^{-1}(\sigma R_{KL} + \tau Z_{LL})$ . We assume that  $(dg/d\tau) = g_\tau = (\Phi + K)$  is positive, that is, a higher capital tax by the unitary government increases the provision of the state/federal public consumption good. Following a similar procedure, assuming  $dg = 0$ , the first order condition for the optimal provision of  $G$  requires that the capital tax is set so that:

$$\frac{1}{N}dW = E_u \frac{du}{d\tau} \Big|_{dg=0} = -E_G G_\tau - K = -NE_G(\Phi + K) - K = 0, \quad (13)$$

where we define  $NE_G$  to be the ‘federal marginal willingness’ to pay for the federal public consumption good, and  $\frac{dG}{d\tau} = G_\tau = Ng_\tau$ .

Combining the first-order conditions (12) and (13), yields the standard optimality rule for the provision of  $g$  and  $G$  as follows:

$$-E_g = \frac{1}{\left(1 + \frac{\Phi}{K}\right)} = \frac{K}{g_\tau} = -NE_G. \quad (14)$$

That is, the unitary government is indifferent between spending a dollar for the provision of  $g$  or  $G$  when the marginal willingness to pay for the provision of a state’s public consumption good equals the federal marginal willingness to pay for the federal public consumption good. The optimality rule (14) determines the unitary optimal capital tax ( $\tau$ ) so that the Marginal Cost of Public Funds (MCPF), i.e.,  $\frac{K}{g_\tau}$ , equals the marginal willingness to pay for the state public consumption good and the federal marginal willingness to pay for the provision of the federal public consumption good.<sup>23</sup> Thus, equation (14) determines the unitary second-best optimum.<sup>24</sup> We can state:

**Lemma 1.** *At the unitary optimum, the specific capital tax  $\tau$  is set so that the marginal willingness to pay for the state public consumption good equals the federal marginal willingness to pay for the federal public consumption good, which are equal to the MCPF.*

## 5. Capital Taxes and Vertical Fiscal Externality

Next, we consider the ‘vertical fiscal externality’, which is the impact of the tax setting behavior of the representative state government on the provision of the federal public consumption good, i.e.,  $\text{sign}(G_\tau)$  given in equations (5). As reviewed, [Boadway and Keen \(1996\)](#) demonstrate that the sign of the ‘vertical fiscal externality’ is unambiguously negative when taxes are specific and only one factor of production is in variable supply. We

23 The equivalent condition in [Kotsogiannis and Martinez \(2008\)](#), that is, their equation (16), states that the second best unitary optimum tax is such that the sum of the marginal rate of substitution between both the state and the federal public goods and the private good are equal to the MCPF.

24 A similar optimality rule for the provision of  $g$  and  $G$  is obtained when the unitary government, for given rate of the capital tax  $\tau$ , chooses the labor tax  $\sigma$  so as to maximize  $W = Nu$  subject to the consolidated budget constraint in (9). In this case, the optimality rule is  $-E_g = \frac{1}{\left(1 + \frac{\Phi}{K}\right)} = \frac{L}{g_\sigma} = -NE_G$ , where  $\Psi = \Delta^{-1}(-\sigma R_{KK} + \tau R_{KL})$  and  $\frac{dg}{d\sigma} = g_\sigma = \Psi + L$ . The interpretation of this rule is the same as the one for equation (14).

revisit these results in the case of endogenous capital and labor supplies and specific factor taxes. For expositional clarity of the results, we consider the case of a federal economy with two identical states ( $N = 2$ ), for example, Home and Foreign. Thus, we examine the ‘vertical fiscal externality’ due to the tax setting behavior of one state, for example, say  $t^*$  of Foreign, on the provision of the federal public consumption good, and on the welfare level of Home, that is,  $du/dt^*$ .

Using equations (5), the effect of changes in  $t^*$  on  $G$  is given as follows:

$$\frac{dG}{dt^*} = G_{t^*} = S \frac{dL^*}{dt^*} + T \frac{dK^*}{dt^*} = \frac{SR_{L^*K^*}^* + TZ_{L^*L^*}^*}{\Delta^*}. \tag{15}$$

where  $\Delta^* = \Delta$  due to the symmetry assumption and is negative (see equation A.1 in the Appendix). An increase in  $t^*$  by Foreign, on the one hand, causes a capital outflow from the federation that reduces capital tax revenue that is,  $\frac{TZ_{L^*L^*}^*}{\Delta^*} < 0$ , and on the other hand, it causes an ambiguous effect on labor tax revenue depending on the relationship between capital and labor in the production. For example, if  $R_{L^*K^*}^*$  is positive, that is, when  $K$  and  $L$  are complements in production, then the higher labor tax  $t^*$  reduces labor tax revenue, since a capital outflow will reduce employment. Thus, the level of federal public good provision falls, that is,  $G_{t^*} < 0$ . If, however,  $R_{L^*K^*}^*$  is negative, that is,  $K$  and  $L$  are substitutes in production, then a capital outflow increases employment and labor tax revenue. In this case the total effect of an increase in  $t^*$  on  $G$  is ambiguous.

The effect on Home’s welfare due to the change in  $t^*$  is obtained by totally differentiating equation (6) and making use of (15) as follows:<sup>25</sup>

$$E_u \frac{du}{dt^*} = -E_G G_{t^*} = -E_G \frac{(SR_{L^*K^*}^* + TZ_{L^*L^*}^*)}{\Delta^*}. \tag{16}$$

Equations (15) and (16) indicate that in the present context of variable labor and capital factor supplies, even the change of a specific capital tax by one state entails an ambiguous impact on the provision of the federal public good, and on the welfare of the other state that is, the change of a state capital tax entails an ambiguous ‘vertical fiscal externality’. If  $R_{L^*K^*}^*$  is positive, then an increase in  $t^*$  by Foreign entails an unambiguously negative ‘vertical fiscal externality’. This is to say that the non-cooperative equilibrium capital tax  $t^*$  is inefficiently high relative to the cooperative one.<sup>26</sup> If, however,  $R_{L^*K^*}^*$  is negative, that is,  $K$  and  $L$  are substitutes in production, then the change of  $t^*$  by Foreign leads to a positive ‘vertical fiscal externality’ if  $(SR_{L^*K^*}^* + TZ_{L^*L^*}^*) < 0$ , i.e.,  $|SR_{L^*K^*}^*| > TZ_{L^*L^*}^*$ . In this case,

25 Totally differentiating equation (6) with respect to  $t^*$  yields  $E_u \frac{du}{dt^*} = -E_g \frac{dg}{dt^*} - E_G \frac{dG}{dt^*}$ . Also from equations (A.1),  $\frac{dK}{dt^*} = \frac{dL}{dt^*} = 0$ , as a result of which  $\frac{dg}{dt^*} = 0$ . Hence, the result in equation (16).

26 The cooperative equilibrium capital tax rate of each country is obtained by maximizing the two countries joint welfare. To obtain the cooperative capital tax rates  $t$  and  $t^*$  we simultaneously solve the first-order conditions  $E_u \frac{du}{dt} + E_v \frac{dv}{dt} = 0$  and  $E_u \frac{du}{dt^*} + E_v \frac{dv}{dt^*} = 0$ . To compare the non-cooperative equilibrium tax rate with the cooperative one we need to evaluate the signs of  $\frac{du}{dt}$  and  $\frac{du}{dt^*}$  at the noncooperative equilibrium. Note that at the noncooperative equilibrium  $E_u \frac{du}{dt} = E_v \frac{dv}{dt} = 0$ . If  $\frac{du}{dt^*}$  is negative (positive) at the non-cooperative equilibrium, this implies that the slope of the joint welfare function is negative (positive) at the non-cooperative equilibrium. Thus, the noncooperative equilibrium capital tax is inefficiently high (low), that is,  $t^{*non-coop} > (<) t^{coop}$ .

the non-cooperative equilibrium capital tax is inefficiently lower compared to the cooperative one. That is, if the increase in labor tax revenue due to capital outflow is larger than the decrease in capital tax revenue, then the increase in  $t^*$  increases the federal tax revenue. Thus, the higher  $t^*$  has a negative (ambiguous) effect on Home's welfare when  $R_{L^*K^*}^*$  is positive (negative).<sup>27</sup> We state the following Proposition:

**Proposition 2** *An increase in the specific capital tax  $t^*$  by Foreign entails a positive vertical fiscal externality and leads to a higher Home welfare if capital and labor are substitutes in production and the following condition holds:*

$$-\eta_{L^*K^*}^* > \frac{TK^*}{SL^*} (\gamma^* \epsilon_{L^*L^*}^* - \eta_{L^*L^*}^*). \quad (17)$$

**Proof.** *see equations (A.4) in the Appendix. ■*

In equation (17)  $\epsilon_{L^*L^*}^* \left( = \frac{\partial E_{L^*}^*}{\partial L^*} \frac{L^*}{E_{L^*}^*} \right) > 0$  and  $\eta_{L^*L^*}^* \left( = \frac{\partial R_{L^*}^*}{\partial L^*} \frac{L^*}{R_{L^*}^*} \right) < 0$ , respectively are the elasticities of the reservation wage and of the marginal revenue product of labor with respect to employment, and using equation (1),  $0 < \gamma^* \left( = \frac{E_{L^*}^*}{R_{L^*}^*} = 1 - \frac{\sigma^*}{R_{L^*}^*} \right) \leq 1$ , the ratio of the reservation wage to the marginal revenue product of labor is less than 1 in the presence of a labor tax. Also,  $\eta_{L^*K^*}^* \left( = \frac{\partial R_{L^*}^*}{\partial K^*} \frac{K^*}{R_{L^*}^*} \right) \leq 0$ , is the elasticity of the marginal revenue product of labor with respect to capital, and its sign depends on the relationship between capital and labor in the production. A positive 'vertical fiscal externality' is more likely to emerge when labor and capital are 'strong' substitutes, the elasticities of the reservation wage and that of the marginal revenue product of labor with respect to labor are small, and federal labor taxes are high relative to capital taxes. Intuitively, when capital and labor are substitutes in production, then an increase in Foreign's capital tax causes capital outflow from the federation but increases labor supply and employment. If condition (15) holds, then the increase in the wage tax revenue is larger than the decrease in the capital tax revenue, leading to an increase in the level of the federal public good. Thus, taxing labor income is a crucial factor in obtaining the above result.<sup>28</sup>

Next, we examine the issue of vertical fiscal externality in two relevant extensions of our basic model.<sup>29</sup> First, we consider the case where the federal government in addition to capital and labor taxes it also levies a tax on the internationally immobile and in fixed supply factors, hereon a rent-tax. Revenue from this taxation are used to finance the provision of the federal public consumption good. Second, the case where capital is mobile between the two states of the federal economy, but not between the federal economy and the 'rest of

27 Alternatively, a similar effect can be obtained by changing Foreign's labor tax  $s^*$ , treating  $t^*$  constant, on Home's welfare which is given as  $E_u \frac{ds^*}{ds^*} = -E_G G_{s^*} = -E_G \Delta^{*-1} (-SR_{K^*K^*}^* + TR_{K^*L^*}^*)$ . Again, the higher  $s^*$  has a negative (ambiguous) effect on Home's welfare when  $R_{K^*L^*}^*$  is positive (negative).

28 This ambiguity does not exist when tax-bases are not shared between the federal and state governments. That is, if the federal government imposes only a labor tax across all states, while each state imposes only a capital tax, that is,  $T = 0$  and  $s = s^* = 0$ , then a higher  $t^*$  entails a positive (negative) vertical fiscal externality and a positive (negative) effect on Home's welfare when capital and labor are substitute (complement) factors in production, see equations (15) and (16).

29 We acknowledge the anonymous referee's suggestion for these extensions.

the world'. Thus, the overall supply of capital in the federal economy is fixed, and the rate of return to capital is now endogenously determined.

5.1 Taxes on fixed factors income

The Federal government now also taxes, at a rate  $\Theta$ , the income of the internationally immobile and in fixed supply factors of production. The ensuing revenue from rental taxes finances the provision of the federal public good  $G$ . All other features of the two-states, Home and Foreign, federal economy remain the same. The emerging model is given by equations (A.5)–(A.6) in the Appendix. Using these equations, the vertical fiscal externality of changes in  $t^*$  on  $G$  is given as follows:

$$G_{t^*} = [S - \Theta(L^*R_{L^*L^*}^* + K^*R_{K^*L^*}^*)] \frac{dL^*}{dt^*} + [T - \Theta(L^*R_{L^*K^*}^* + K^*R_{K^*K^*}^*)] \frac{dK^*}{dt^*}, \tag{18}$$

where by equations (A.2),  $\frac{dK^*}{dt^*} < 0$ , and  $\frac{dL^*}{dt^*} > (<)0$  depending on whether  $R_{K^*L^*}^* < 0(> 0)$ . Let  $R_{K^*L^*}^* < 0$ , that is,  $L$  and  $K$  are substitutes in production. Then, the two bracketed terms in the right-hand-side of the equation are positive. Thus, the sign of the vertical fiscal externality of changes in  $t^*$  is ambiguous since  $\frac{dK^*}{dt^*} < 0$ , and  $\frac{dL^*}{dt^*} > 0$ . Intuitively, the increase in the capital tax that reduces capital, on the one hand, increases employment. This reduces wages and the return to capital, both of which increase the income of the fixed factors. Thus, as result of the increase in employment, revenues from wage and rent taxes increase. On the other hand, the reduction in the stock of capital, increases wages and the return to capital which reduces rents. Thus as a result of the reduction in the stock of capital, tax revenue from taxing capital and rents are reduced. Therefore, the effect of an increase in capital tax on federal revenue is ambiguous. When  $R_{K^*L^*}^* > 0$ , the ambiguity remains. In this case, while the increase in capital taxes reduce employment, and thus capital and labor tax revenue decline, the effect on rents is ambiguous. Thus, when the federal government taxes rents the sign of the vertical fiscal externality is ambiguous even when capital and labor are complements in production. For the sign of the vertical fiscal externality to be negative, tax revenue from rents must be sufficiently small.<sup>30</sup> In the special case where only capital supply is variable, due to international mobility, while all other factors, including labor, are in fixed supply, equation (18) reduces to  $G_{t^*} = (T - \Theta K^* R_{K^*K^*}^*) \frac{dK^*}{dt^*}$ , indicating, an unambiguously negative vertical fiscal externality due to changes in  $t^*$ . In this case, our model reproduces the standard literature result of a negative vertical fiscal externality when capital taxes are specific.<sup>31</sup>

5.2 Capital mobility only within the federation

Next, consider the case where capital is mobile only within the federation. That is, the stock of capital in the federation is fixed. We will examine whether now a horizontal fiscal

30 In case where the rent tax is levied by the state governments for the financing of the state public consumption goods, as in Keen and Kotsogiannis (2002), then the vertical fiscal externality is the same as the one given in equation (15).

31 Since the change in  $t^*$  does not affect the levels of  $K$  and  $L$ , the effect on Home's welfare due to the change in  $t^*$  depends only on the induced changes in the level of the federal public consumption good, that is,  $E_u \frac{dU}{dt^*} = -E_G G_{t^*}$ .

externality also emerges, and if so, we determine its sign under the assumptions of the model. Now, the capital market equilibrium condition (2) and the corresponding capital market equilibrium condition for the second state, are replaced by the equilibrium condition for the ‘federal’ capital market, which is as follows,

$$R_K(K, L) - \tau = R_{K^*}^*(K^*, L^*) - \tau^*. \quad (19)$$

The state labor markets equilibrium conditions, that is, equilibrium conditions (1) and the corresponding one for Foreign, and the federal capital market equilibrium condition (19) are used to determine the effects of changes in capital and labor taxes on the supplies of the two factors, that is,  $L$ ,  $L^*$  and  $K(K^*)$ , in the two states. The relevant comparative statics are presented in Appendix by equations (A.7).

Totally differentiating the budget constraints (3) and (4) and using equations (A.7) from the Appendix, we obtain the following results:

$$g_{t^*} = -\Delta_1^{-1} Z_{L^*, L^*}^* (sR_{LK} + tZ_{LL}), \quad (20)$$

$$G_{t^*} = S\Delta_1^{-1} (Z_{LL}R_{L^*K^*}^* - Z_{L^*, L^*}^* R_{LK}). \quad (21)$$

Equation (20) indicates that a change in Foreign’s capital tax  $t^*$ , affects Home’s provision of the public consumption good  $g$ . Specifically, when capital and labor are substitutes in production, the emerging horizontal fiscal externality can be either positive or negative. That is, when  $R_{LK} < 0$ , we get  $g_{t^*} \geq 0$ . When they are complements, the emerging horizontal fiscal externality is positive, that is,  $R_{LK} > 0$  gives  $g_{t^*} > 0$ . The horizontal fiscal externality remains even when the two states are symmetric.

Equation (21) indicates that unless capital and labor are of different relationship in production in the two states, for example,  $R_{LK} > 0$  and  $R_{L^*K^*}^* < 0$ , the resulting vertical fiscal externality is of ambiguous sign, that is,  $G_{t^*} \geq 0$ . When the two states are symmetric, that is,  $Z_{LL} = Z_{L^*, L^*}^*$  and  $R_{LK} = R_{L^*K^*}^*$ , and since by assumption  $dK = -dK^*$ , then the resulting vertical fiscal externality is zero.<sup>32</sup>

The previous analysis considers the two polar cases where (i) the federal economy is small in world capital markets, and capital is internationally mobile, (ii) capital is only mobile within the federation. We find that in the former case there is vertical but not horizontal fiscal externalities, while in the later, when states are symmetric, there is horizontal but not vertical fiscal externalities. When the federal economy is large in world capital markets, capital is internationally mobile, and a state increases its capital tax, then, part of its capital

32 Keen and Kotsogiannis (2002) develop a two-period model of a federal economy with identical states, where capital is only inter-state mobile and taxed by both state and federal governments. Furthermore, rents are taxed by the state governments in order to finance the provision of state public consumption goods. They examine the conditions under which vertical or horizontal fiscal externalities dominate each other, as a result of changes in state capital taxes. One thing they emphasize is that the horizontal fiscal externalities disappear when, due to changes in state capital taxes, there are no changes in the rate of return to capital across states and no horizontal movements of capital. This case resembles our analysis where there are no induced horizontal fiscal externalities since the federal economy is assumed small in world capital markets. Thus, a change in  $t^*$  induces capital flows to/from the ‘rest of the world’ while the supply of capital in Home remains unchanged.

moves to the other states and to the rest of the world. In this case, we expect both types of fiscal externalities to exist.<sup>33</sup> However, when states are symmetric the vertical fiscal externality is expected to be smaller than that in case (i), but still of ambiguous sign. The horizontal fiscal externality is expected to be smaller than that in case (ii), but still of ambiguous sign.

### 6. Federal Tax-Transfers with Centralized Leadership

We now address the issue of resource transfers between the different levels of government. Previous studies using models with one factor of production in variable supply, conclude that with specific taxes and under centralized leadership, resulting to negative vertical fiscal externality, the federal government sets a negative specific tax on the rate of return to the factor in variable supply, which can lead to bottom-up transfers (e.g., among others [Boadway and Keen 1996](#)). Subsequent studies demonstrate that with ad valorem taxation the above results can be reversed if the demand for the factor in variable supply is inelastic (e.g., [Kotsogiannis and Martinez 2008](#); [Karakosta 2010](#); [Sas 2017](#)). We revisit these results in the case of variable labor and capital supplies and specific factor taxes. We consider a two-stage game where the two levels of government pursue separately the setting of fiscal policy, for example, the choice of capital taxes  $t$  and  $T$ , treating as exogenous the rates of federal and state labor taxes  $S$  and  $s$ . We assume that the federal government holds a first mover’s advantage vis-a-vis all states, that is, centralized leadership. In the first stage the federal government chooses the capital tax rate  $T$  and the level of the vertical transfer  $F$  in order to maximize the federal economy’s cumulative welfare level  $W = Nu$ . In the second stage a typical state government chooses its own specific capital tax  $t$ , in order to maximize its own welfare  $u$ , given  $T$  and  $F$ . As standard practice calls for, the subgame perfect equilibrium is solved backwards.

Starting with the second stage of the game, the optimal specific capital tax for a typical state is determined by totally differentiating the income-expenditure identity (6) and setting the first-order-condition equal to zero. Doing so we obtain:

$$E_u \frac{du}{dt} = H_t = -E_g \frac{dg}{dt} - K = -E_g(K + \alpha) - K = 0 \Rightarrow t^o = t^o(T, F). \tag{22}$$

Changes in  $t^o$  due to changes in  $T$  and  $F$  are obtained by totally differentiating the first-order-condition  $H_t = 0$  with respect to  $t$ ,  $T$ , and  $F$  to get:

$$\frac{dt}{dT} = t_T = -\frac{H_{tT}}{H_{tt}} \text{ and } \frac{dt}{dF} = t_F = -\frac{H_{tF}}{H_{tt}} < 0, \tag{23}$$

where  $H_{tt}$ , and  $H_{tT}$  are defined in equations (A.8) in the Appendix, which provide details of these derivations. The second-order condition for welfare maximization requires that  $H_{tt} < 0$ , and by the assumptions of our model  $H_{tF} = -(K + \alpha)E_{gg} < 0$ . Thus,  $t_F < 0$ , implying that a ‘top-down transfer’ reduces the state’s capital tax rate. The sign of  $t_T$  is ambiguous.

In the first stage of the game the federal government chooses  $T$  and  $F$  in order to maximize  $W = Nu$ , accounting now for the adjustments in the state capital tax  $t$  due to the

33 Such an extension is interesting but it is left for future research.

changes in  $T$  and  $F$ . Using equation (6), the first-order conditions for the optimum  $T$  and  $F$  are given as follows:

$$\frac{dW}{dT} = 0 \Rightarrow \frac{1}{N} W_T = E_u \frac{du}{dT} = 0 \Rightarrow KE_g - NE_G [G_t(1 + t_T) + K] = 0, \quad (24)$$

$$\frac{1}{N} W_F = E_u \frac{du}{dF} = 0 \Rightarrow (-E_g + NE_G) - NE_G G_t t_F = 0. \quad (25)$$

Equations (A.9)–(A.13) in the Appendix provide the details of this derivation.<sup>34</sup>

The first-order condition for the optimum  $T$ , can be further written as:

$$-E_g = -NE_G \left[ 1 + \frac{G_t(1 + t_T)}{K} \right]. \quad (26)$$

Equation (26) indicates that when  $G_t(1 + t_T) < 0 (> 0)$ , then the marginal willingness to pay for the provision of the state public consumption good is lower (higher) than that for the federal public consumption good, that is,  $-E_g < -NE_G (-E_g > -NE_G)$ .<sup>35</sup>

The first-order condition for the optimum  $F$ , in equation (25), determines the direction of the inter-governmental transfer  $F$ , in the presence of the ‘vertical fiscal externality’, variable labor and capital supplies, and specific factor taxes. This condition can be further written as:

$$-E_g = -NE_G(1 - G_t t_F) \quad (27)$$

Since  $t_F$  is negative, equation (27) indicates that the direction of the transfer between the state and the federal government depends on the sign of  $G_t$ . The sign of  $G_t$ , depends on the relationship between  $K$  and  $L$  in production, see discussion of equations (15) and (16). If  $G_t > 0$ , then,  $-E_g > -NE_G$ . That is, the increase in the federal labor income tax revenue due to the increase in the state capital tax is larger than the decrease in federal capital income tax revenue, leading to an increase in the federal public good, making its level too high. In this case, the transfer should go from the federal to the state government, that is, a ‘top-down transfer’. Intuitively, the transfer should go from the government providing the public good with lower marginal willingness to pay for its provision, in this case being the federal government, to that providing the public good with higher marginal willingness to pay for its provision, in this case being the state or regional government. The transfer will affect the state capital tax and the vertical externality. Since  $G_t t_F < 0$ , the transfer reduces the vertical externality. If  $G_t < 0$ , then,  $-E_g < -NE_G$ . In this case the increase in the state capital tax reduces the provision of the federal public good, making its level too low. The transfer should go from the state to the federal government, that is, a ‘bottom-up transfer’.

Combining the first-order conditions (24) and (25) it is easily shown that the optimal federal capital tax ( $T^{\text{op}}$ ) is determined by setting  $G_t = 0$  and is given as follows:

34 In deriving the optimality conditions (24) and (25) we make use of equations (5) and of the first-order condition  $H_t = 0 \Rightarrow E_g(K + \alpha) = -K$ .

35 In Kotsogiannis and Martinez (2008), the equivalent condition, that is, their equation (22), is stated in terms of MCPFs for the state and federal governments.

$$T^{\text{op}} = -SR_{LK}Z_{LL}^{-1} = -\frac{SL\eta_{LK}}{K(\gamma\epsilon_{LL} - \eta_{LL})}, \quad (28)$$

where the elasticities,  $\eta_{LK}$ ,  $\epsilon_{LL}$  and  $\eta_{LL}$ , and the ratio  $\gamma$  are equivalently defined to the corresponding terms in optimality condition (17) due to the assumption of symmetric states. Observing [equation \(28\)](#) we can conclude the following. First,  $T^{\text{op}}$  is a capital tax (subsidy) when capital and labor are substitutes (complements) in production. Second, the optimal federal capital tax or subsidy rate is high if  $S$ ,  $L$ , and  $\eta_{LL}$ , are high and  $K$ ,  $\eta_{LK}$ ,  $\epsilon_{LL}$ , and  $\gamma$  are low. When the federal government chooses simultaneously its capital tax and transfer, then conditions (24) and (25) replicate the unitary second-best optimum in (14) since  $-E_g = -NE_G$ . The following Proposition summarizes the above results.

**Proposition 3** *Under centralized leadership the federal government achieves the second-best optimum by setting (i) a capital subsidy and bottom-up transfers when capital and labor are complements in production, and (ii) a capital tax and, possibly, top-down transfers when capital and labor are substitutes in production.*

Consider the case where capital and labor are substitutes in production, thus, the optimal policy calls for a federal capital tax. If, for example, the federal public good is *worthless*, that is,  $G = 0$ , then [equation \(4\)](#) indicates that  $F > 0$ , implying a top-down transfer.<sup>36</sup> [Koethenburger \(2008\)](#) in a context of no tax base co-sharing and with capital and labor in variable supplies but independent in production, shows that while the federal government improves efficiency by precommitting to its policy, that is, centralized leadership, is not able to replicate the unitary government's second-best optimum. In this article, we show that with co-sharing of tax bases and variable supplies of capital and labor, the federal government by precommitting to its policy achieves the unitary government's second-best optimum.<sup>37</sup> In the present framework, it is the relationship in production between capital and labor which is crucial for the sign of the federal capital tax and direction of the federal-state resource transfers.

## 7. Concluding Remarks

We reconsider the 'vertical fiscal externality' and the direction of federal-state transfers, and their relationship in a model of a multistates federal economy with federal-cum-state specific factor taxes and variable supplies of labor and capital in production. The supply of labor is variable due to endogenous labor-leisure decision and the supply of capital is variable due to its international mobility. We show that, first, when capital and labor are substitutes in production, then, (i) the optimal policy for a representative state calls for a capital and a labor tax, and (ii) the 'vertical fiscal externality' can be reversed from negative, implying inefficiently high noncooperative specific capital taxes, to positive, implying inefficiently low noncooperative specific capital taxes. Second, with co-sharing of tax bases and variable supplies of capital

36 For the case of worthless federal public goods (see, among others, [Boadway and Keen 1996](#); [Kotsogiannis and Martinez 2008](#)).

37 An equivalent result is obtained in a two-stage game where in the first stage the federal government chooses the labor tax  $S$ , instead of the capital tax  $T$ , and the level of the vertical transfer  $F$ . In the second stage a typical state government chooses its own specific labor tax  $s$ , instead of the capital tax  $t$ , given  $S$  and  $F$ .



and labor, the federal government by precommitting to its policy achieves the unitary government's second-best optimum. Specifically, with an exogenous labor tax, when capital and labor are complements in production, then, they are substitutes in production, the second-best optimum is achieved with a capital tax and possible 'top-down transfers'.

In our model of a small open federation with international capital mobility, horizontal fiscal externalities do not exit. If, however, capital is only mobile within the federation, then both horizontal and vertical externalities exist. The sign of the vertical fiscal externality is generally ambiguous, while that of the horizontal fiscal externality is positive when capital and labor are complements in production, and ambiguous when they are substitutes. When states are symmetric, the horizontal fiscal externality remains while the vertical one ceases to exist.

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## Appendix

### Effects of changes in $\tau$ and $\sigma$ on $K$ and $L$

Totally differentiating the equilibrium conditions (1) and (2) we obtain:

$$\begin{bmatrix} Z_{LL} & -R_{LK} \\ R_{KL} & R_{KK} \end{bmatrix} \begin{bmatrix} dL \\ dK \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} d\tau + \begin{bmatrix} -1 \\ 0 \end{bmatrix} d\sigma, \quad (\text{A.1})$$

from which we obtain:

$$\begin{aligned} \frac{dL}{d\tau} &= \Delta^{-1} R_{LK}, & \frac{dL}{d\sigma} &= -\Delta^{-1} R_{KK}, \\ \frac{dK}{d\tau} &= \Delta^{-1} Z_{LL}, & \frac{dK}{d\sigma} &= \Delta^{-1} R_{KL}, \end{aligned} \quad (\text{A.2})$$

where  $\Delta = R_{KK}(Z_{LL} + R_{LK}R_{KK}^{-1}R_{KL})$  is negative, and  $Z_{LL} = E_{LL} - R_{LL}$  is positive by the properties of the  $E(\cdot)$  and  $R(\cdot)$  functions. The sign of  $R_{LK} = \partial R_L / \partial K$  depends on the relationship between  $K$  and  $L$  in production. The total differentiation of the equilibrium condition (2) gives  $(dK/dL) = -R_{KK}^{-1}R_{KL}$ . If  $R_{KL} < (>)0$ , then  $K$  and  $L$  are substitutes (complements) in production. In the two factors model  $R_{KL} < 0$ , that,  $K$  and  $L$  are substitutes in production. In a model with more than two factors, however, we can have  $K$  and  $L$  being complements in production, that is,  $R_{KL} > 0$ . Note that  $\frac{dL}{dF} = \frac{dK}{dF} = 0$ .

### Comparative statics results for equations (5)

Total differentiation of the state and federal governments budget constraints with respect to the capital taxes ( $t$  and  $T$ ) gives:

$$\begin{aligned}
 g_t &= K + s \frac{dL}{dt} + t \frac{dK}{dt}, \quad g_T = s \frac{dL}{dT} + t \frac{dK}{dT}, \quad G_t = \left( S \frac{dL}{dt} + T \frac{dK}{dt} \right), \quad \text{and} \\
 G_T &= N \left( K + S \frac{dL}{dT} + T \frac{dK}{dT} \right).
 \end{aligned}
 \tag{A.3}$$

Using the results in (A.2) where  $\tau = t + T$ , we obtain the expressions in the text.

**Comparative statics results for equation (17)**

The sign  $\left(\frac{dG}{dt}\right)$  and sign  $\left(\frac{du}{dt}\right)$ , in equations (15) and (16) respectively, depend on the sign  $(SR_{L^*K^*}^* + TZ_{L^*L^*}^*)$  in the right-hand-side of these equations. Recall that by equations (A.1), the determinant  $\Delta^*$  is negative. Then, straightforward algebra produces the following result:

$$\begin{aligned}
 SR_{L^*K^*}^* + TZ_{L^*L^*}^* &= T \left( \frac{S}{T} \frac{\partial R_{L^*}^*}{\partial K^*} \frac{K^* R_{L^*}^*}{R_{L^*}^* K^*} + \frac{\partial E_{L^*}^*}{\partial L^*} \frac{L^* E_{L^*}^*}{E_{L^*}^* L^*} - \frac{\partial R_{L^*}^*}{\partial L^*} \frac{L^* R_{L^*}^*}{R_{L^*}^* L^*} \right) = \\
 &\frac{TR_{L^*}^*}{L^*} \left( \frac{SL^*}{TK^*} \eta_{L^*K^*}^* + \gamma \epsilon_{L^*L^*}^* - \eta_{L^*L^*}^* \right),
 \end{aligned}
 \tag{A.4}$$

then, the condition (17) emerges.

**Taxes on fixed factors income**

In the case where the federal government taxes rents, the model of the two-states open federal economy consists of equations (1)–(3), and the following federal government budget constraint, and the representative state’s income-expenditure identity.

$$\begin{aligned}
 G &= S(L + L^*) + T(K + K^*) \\
 &\quad + \Theta \{ [R(\cdot) - LR_L(\cdot) - KR_K(\cdot)] + [R^*(\cdot) - L^*R_{L^*}^*(\cdot) - K^*R_{K^*}^*(\cdot)] \},
 \end{aligned}
 \tag{A.5}$$

$$E(g, G, L, u) = R(\cdot) - \tau K - \sigma L - \rho k - \Theta [R(\cdot) - LR_L(\cdot) - KR_K(\cdot)].
 \tag{A.6}$$

Differentiating the federal budget constraint with respect to  $t^*$ , and since  $L$  and  $K$  are not affected by changes in  $t^*$ , gives equation (18) in the text.

**Capital mobility only within the Federation**

The equilibrium conditions for the labor and capital markets are now given by:

$$\begin{aligned}
 E_L(g, G, L, u) &= R_L(K, L) - \sigma, \\
 E_{L^*}^*(g^*, G, L^*, u^*) &= R_{L^*}^*(K^*, L^*) - \sigma^*, \\
 R_K(K, L) - \tau &= R_{K^*}^*(K^*, L^*) - \tau^*.
 \end{aligned}$$

Totally differentiating the above equilibrium conditions with respect to the state capital taxes  $(t, t^*)$  we obtain:

$$\begin{aligned} \frac{dL}{dt} &= \Delta_1^{-1} Z_{L^*L^*}^* R_{LK}, & \frac{dL}{dt^*} &= -\Delta_1^{-1} Z_{L^*L^*}^* R_{LK}, \\ \frac{dK}{dt} &= \Delta_1^{-1} Z_{LL} Z_{L^*L^*}^* R_{LK}, & \frac{dK}{dt^*} &= -\Delta_1^{-1} Z_{LL} Z_{L^*L^*}^* R_{LK}, \end{aligned} \quad (\text{A.7})$$

where  $\Delta_1 = Z_{LL} Z_{L^*L^*}^* (\tilde{R}_{KK} + \tilde{R}_{KK}^*) < 0$ ,  $\tilde{R}_{KK} = R_{KK} + R_{KL} Z_{LL}^{-1} R_{LK}$ ,  $\tilde{R}_{K^*K^*}^* = R_{K^*K^*}^* + R_{K^*L^*}^* Z_{L^*L^*}^{*-1} R_{L^*K^*}^*$ , both being negative by the strict concavity of the  $R(\cdot)$  and  $R^*(\cdot)$  functions with respect to factor supplies.

### Derivation of equations (23)

Totally differentiating the first-order condition  $H_t = 0$  with respect to  $t$ ,  $T$  and  $F$  we get:

$$dH_t = 0 = H_{tt}dt + H_{tT}dT + H_{tF}dF = 0,$$

from which equations (23) in the text are obtained, and

$$\begin{aligned} H_{tt} &= \left\{ \begin{aligned} &-(K + \alpha)E_{gg}g_t - E_g\alpha_t - (E_g + 1)\frac{dK}{dt} = \\ &= -(K + \alpha)E_{gg}(K + \alpha) - E_g\Delta^{-1}Z_{LL} - (E_g + 1)\Delta^{-1}Z_{LL} \end{aligned} \right\} \\ H_{tT} &= \left\{ \begin{aligned} &-(K + \alpha)E_{gg}g_T - E_g\left(\frac{dK}{dT} + \alpha_T\right) - \frac{dK}{dT} = \\ &= -(K + \alpha)E_{gg}\alpha - (1 + E_g)\Delta^{-1}Z_{LL} \end{aligned} \right\} \\ H_{tF} &= -(K + \alpha)E_{gg}g_F + E_g\left(\frac{dK}{dF} + \alpha_F\right) - \frac{dK}{dF} = -(K + \alpha)E_{gg} < 0, \end{aligned} \quad (\text{A.8})$$

where  $\alpha_F = \frac{dz}{dF} = 0$ ,  $\frac{dK}{dF} = 0$ ,  $\frac{dK}{dT} = \frac{dK}{dT} = \Delta^{-1}Z_{LL}$ , and assuming that third derivatives are zero  $\alpha_t = \frac{dz}{dt} = \Delta^{-1}Z_{LL}$  and  $\alpha_T = \frac{dz}{dT} = 0$ .

### First-stage comparative statics: Derivation of equations (24) and (25)

The first-order condition (24) can be written as:

$$\frac{1}{N}W_T = E_u \frac{du}{dT} = 0 \Rightarrow -E_g \frac{dg}{dT} - E_G \frac{dG}{dT} - (1 + t_T)K = 0, \quad (\text{A.9})$$

where by totally differentiating the two government budget constraints, equations (3) and (4), with respect to  $T$ , accounting for the induced adjustments in  $t$  due to changes in  $T$ , we have:

$$\begin{aligned} \frac{dg}{dT} &= s \left( \frac{dL}{dT} + \frac{dL}{dt} t_T \right) + t \left( \frac{dK}{dT} + \frac{dK}{dt} t_T \right) + K t_T = g_T + g_t t_T, \text{ and} \\ \frac{dG}{dT} &= N \left( S \frac{dL}{dT} + S \frac{dL}{dt} t_T + T \frac{dK}{dT} + T \frac{dK}{dt} t_T + K \right) = G_T + N G_t t_T. \end{aligned} \quad (\text{A.10})$$

Substituting equations (A.10) into equation (A.9) and recalling that  $H_t = 0 \Rightarrow -E_g(\alpha + K) = K$ , gives equation (24) in the text. Similarly, the first-order condition (25) can be written as:

$$\frac{1}{N} W_F = E_u \frac{du}{dF} = 0 \Rightarrow -E_g \frac{dg}{dF} - E_G \frac{dG}{dF} - K t_F = 0. \quad (\text{A.11})$$

Totally differentiating the two government budget constraints, equations (3) and (4), with respect to  $F$ , and accounting for the induced adjustments in  $t$  due to changes in  $F$ , we have:

$$\begin{aligned} \frac{dg}{dF} &= s \left( \frac{dL}{dF} + \frac{dL}{dt} t_F \right) + t \left( \frac{dK}{dF} + \frac{dK}{dT} t_F \right) + K t_F + 1 = g_F + g_t t_F, \text{ and} \\ \frac{dG}{dF} &= N \left( S \frac{dL}{dF} + S \frac{dL}{dt} t_F + T \frac{dK}{dF} + T \frac{dK}{dT} t_F - 1 \right) = G_F + N G_t t_F. \end{aligned} \quad (\text{A.12})$$

where  $\frac{dL}{dF} = 0$  and  $\frac{dK}{dF} = 0$ . Substituting equations (A.12) into equation (A.11) we obtain:

$$(-E_g + N E_G) - [(1 + E_g)K + E_g \alpha + E_G N G_t] t_F = 0, \quad (\text{A.13})$$

and recalling that  $H_t = 0 \Rightarrow E_g \alpha = -(1 + E_g)K$ , gives equation (25) in the text.

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