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Horizontal versus vertical fiscal equalization: the assignment problem

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Abstract We analyze a model in which the central government can establish a vertical equalization scheme, whereas the regional governments can set up a horizontal equalization scheme. The two levels of government decide in different chronological order. It turns out that, regardless of the timing, the central government always prevails—horizontal equalization does not take place. However, the subgame-perfect equilibrium is only Pareto-efficient, if the central government acts as a Stackelberg leader. Moreover, if the goal of achieving equality in living conditions across the regions is pursued in the model economy, the only suitable candidate for reaching this goal is vertical equalization.

Keywords Federations · Fiscal equalization · Labor mobility · Regional public goods

JEL Classification H23 · H41 · H77

1 Introduction

"One perennial issue in fiscal federalism is the so-called assignment problem, that is, the problem of assigning governmental functions to different levels of government" (Boadway et al. 2008, p. 2285). In this context, a conclusive answer has not yet been found to the question of which level of government should be assigned with the task of equalizing the financial capacities of regions in a federation by means of transfers

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(horizontal versus vertical equalization) or whether both levels should contribute. The purpose of this paper is to contribute to solving this problem.

The purpose of an *efficiency*-oriented fiscal equalization scheme is to achieve an efficient distribution of the entire mobile population across the different regions by means of income transfers from one region to another. This was the focus of attention in the literature in the past.

Moreover, fiscal equalization between the regions of a federation also serves as a tool to achieve the goal of *equal standards of living* in the entire federation which is enshrined in the constitutions of various federations: For example, the Canadian constitution in section 36(1) states: "…, Parliament and the legislatures, together with the government of Canada and the provincial governments, are committed to (a) promoting equal opportunities for the well-being of Canadians; (b) …". Similarly, Art. 72(2) of the German Basic Law demands that the central (federal) government shall establish and maintain "equal living conditions in the federal territory."

The debate on efficiency-oriented fiscal equalization began with the contributions of Buchanan and Goetz (1972) and Flatters et al. (1974). In the model of Flatters et al., the workers of the federation can move from one region to another without any barriers to mobility. To reach, in migration equilibrium, an efficient distribution of the workers of the federation, an interregional income transfer is generally necessary. In Flatters et al., effecting this transfer is the task of the central government within the framework of a vertical equalization scheme. The problem of assigning this task to the central government is not discussed. In 1990, Myers demonstrated that vertical equalization is not required in this model to achieve an efficient migration equilibrium—horizontal equalization suffices. "While it is true that interregional transfers are generally required to achieve a Pareto optimum, it is also true that the Nash competing regional authorities will make these transfers in their own interest" (Myers 1990, p. 114). The debate was intensified by Krelove (1992), who developed a model in which, due to tax exporting, a separate fiscal equalization scheme is not necessary at all. In 1993, Mansoorian and Myers extended Myers' model (1990) with a barrier to mobility in the form of varying degrees of attachment to home. They showed that even in such an extended model, a horizontal equalization scheme is sufficient to guarantee Pareto-efficiency (Mansoorian and Myers 1993, p. 128). Wellisch (1994) supplemented the model of Mansoorian and Myers (1993) with spillover effects caused by the provision of regional public goods. Wellisch demonstrated that if interregional transfers are made either by regional governments or by a central authority, imperfect mobility and spillover effects may allow the resulting Nash equilibria to provide efficient migration equilibria, but the provision of public goods is inefficient.

In the contributions of Caplan et al. (2000), Köthenbürger (2007), Caplan and Silva (2011) and Duran-Vigneron (2012), the central government decides on the interregional transfer and the regional governments decide on the quantity of public goods they provide. The innovation introduced in this context is that timing issues are considered, and decisions of the regional governments, on the one hand, and of the central government, on the other hand, are no longer taken at the same time.

This paper extends the literature as follows. Both horizontal equalization and vertical equalization are considered. Decisions concerning fiscal equalization are no longer taken *either* by the regional governments *or* by the central government, but instead, the regional governments *and* the central government can plan a fiscal equalization system autonomously. Further, the timing of decisions is considered, and different orders of the regional versus the central governments' decisions are studied. This allows us to analyze the assignment problem, i.e., which level of government should carry out the equalization transfers (horizontal versus vertical equalization) or whether both levels should contribute.

The paper is organized as follows. In Sect. 2, we present the model and the conditions for Pareto-efficiency. In Sect. 3, we derive the equilibria resulting from different timings of the decisions of the governments and examine their efficiency under centralized and decentralized leadership. In Sect. 4, we discuss the problem of achieving equal standards of living throughout the federation. The final Sect. 5 provides conclusions.

2 The model and Pareto-efficiency

Let there be a federation consisting of a central government and two regions, each with a regional government. The total population N is normalized to unity ($N = 1$) and can migrate between the two regions, with the population of region i ($i \in \{1, 2\}$) denoted by N^i. In each region, competing firms produce a numéraire good G. The quantity G^i of the goods produced in each region depends on the fixed endowment of region i with an immobile production factor $\bar{L}^i$ (e.g., land) owned by regional government i.[1] Moreover, it is assumed that each resident of a region supplies one unit of labor perfectly inelastically in that region. The production thus also depends on the region's population N^i which is variable. Production G^i can be described using the following production function:

$$G^i = F^i\left(N^i, \bar{L}^i\right) \tag{1}$$

which has the usual neoclassical characteristics.

Production G^i may be used for private consumption as well as for the provision of a regional public good. We assume that all inhabitants of region i have the same *per capita* private consumption x^i. The amount of the regional public good provided in region i is labeled as Z^i. Thus:

$$G^i = N^i x^i + Z^i. \tag{2}$$

The competitive firms pay a wage w^i equal to the respective marginal product of labor[2]:

$$w^i = F^i_N(N^i, \bar{L}^i). \tag{3}$$

[1] This simplification will be commented on further below.

[2] Partial differentiation of a function f w.r.t. x is denoted with a subscript throughout this paper, *i.e.*, $f_x \equiv \partial f / \partial x$.

The firms pay the following rent R^i to the land owners, i.e., the regional governments

$$R^i = F^i - N^i w^i. \tag{4}$$

The regional governments use these earnings to finance the provision of the regional public goods. If a surplus remains, it is distributed equally among the inhabitants of the respective region. If, in contrast, a deficit arises it is financed by a tax levied *per capita* and equal for inhabitants of the respective region. Both situations may be summarized by setting the *per capita* subvention or tax, respectively, to be $\tau^i(\tau^i > 0$, or $\tau^i < 0$, as the case may be; if $\tau^i = 0$, we have a Henry George world).

Based on the above assumptions, the *per capita* consumption in region i amounts to:

$$x^i = w^i + \tau^i. \tag{5}$$

Concerning fiscal equalization, we make the following assumptions: Region i carries out a voluntary transfer S^{ij} to the other region j (horizontal equalization). As commonly done in the literature,[3] we assume that the central government finances a transfer T^2 to region 2 by a corresponding negative transfer T^1 from region 1. The central government's budget constraint is thus: $-T^1 = T^2$ (for simplicity, the superscripts will be omitted in the following and the transfer T^2 to region 2 will be denoted as T).

The budget constraint of regional government i is given by:

$$R^i + S^{ji} = S^{ij} - (-1)^i T + Z^i + \tau^i N^i, \tag{6}$$

which implies that the central government obtains the funds for the transfer T from regional government 1 and then forwards these funds to regional government 2.

Using relation (6) and Eq. (4), the *per capita* consumption (5) can be rewritten as

$$x^i = \frac{1}{N^i}\left[F^i - S^{ij} + S^{ji} + (-1)^i T - Z^i\right]. \tag{7}$$

A further remark concerning the earnings of the regional governments is perhaps required at this point. The assumption that the regional governments own the immobile production factor $\bar{L}^i$ of the respective regions and thus earn the rent R^i seems to be critical at first sight. However, a closer look shows that this is not the case. An alternative assumption could be that the immobile production factor $\bar{L}^i$ is privately owned and that the rents R^i are earned by the residents. Along these lines, Krelove (1992) assumes that the land of a federation is distributed equally among all inhabitants of the federation. Thus, with the corresponding tax exporting he ensures that the necessary monetary flows occur without fiscal equalization. In contrast, Wellisch assumes that the land of each region is equally distributed among the inhabitants of the respective region (Wellisch 1994, p. 173). Based on this assumption, an efficient distribution of

[3] Flatters et al. 1974, p. 104, Wellisch 1994, p. 178, Caplan et al. 2000, p. 270, Köthenbürger 2007, p. 485, Caplan and Silva 2011, p. 328, Duran-Vigneron 2012, p. 106/107.

the population of the federation among the regions cannot be achieved without fiscal equalization. However, it should be noted that the assumption of Wellisch (1994) leads to the following problem: If, after a change in fiscal equalization, citizens move from one region to the other and take their property rights with them, then there are again inhabitants of a certain region who own land in the respective other region. This is in contradiction with the initial assumption of all land of a region being the exclusive property of the inhabitants of that region. Wellisch solves this problem by assuming that the property rights regarding land are conferred only *after* the completion of migration (Wellisch 1994, p. 174, FN 5). However, the same result is also obtained if one assumes that the land is owned by the regional governments (Mansoorian and Myers 1993, p. 122),[4] as also assumed in the present model.

In the present model, the residents of each region derive utility from the consumption of both the private good x^i and the public good Z^i provided in their respective region. Furthermore, there is a varying degree of attachment to home (Mansoorian and Myers 1993, 1997). The extent of a resident's attachment to region 2 is described by coefficient n ($n \in [0, 1]$), with a higher value of n indicating a higher level of attachment. The utility function of a resident with coefficient n can therefore be written in total as

$$V^n = \begin{cases} U^1\left(x^1, Z^1\right) + k\,(1-n)\,, & \text{if living in region 1,} \\ U^2\left(x^2, Z^2\right) + kn, & \text{if living in region 2,} \end{cases} \tag{8}$$

with $k > 0$ expressing the intensity of the attachment to a region.

It is noted that spillover effects are expressly disregarded, here. In Anetsberger and Arnold (2017) spillover effects are considered, and it is shown that these do not affect the main conclusion of the present paper, irrespective of the way in which fiscal equalization is managed.

Function U^i is concave in all variables.[5] Since the population is mobile, an equilibrium in the population distribution across regions and a corresponding individual who is indifferent regarding both regions exist.[6] For this individual, the following therefore holds:

$$U^1\left(x^1, Z^1\right) + k\left(1-n^*\right) = U^2\left(x^2, Z^2\right) + kn^*. \tag{9}$$

Individuals for whom $n > n^*$ live in region 2, whereas individuals for whom $n < n^*$ live in region 1. Thus, n^* at the same time represents the number of inhabitants of region 1, i.e., $n^* = N^1$ and $1 - n^* = N^2$ (Mansoorian and Myers 1997, p. 269). Using

[4] A similar result is obtained by Boadway and Keen (1996, p. 140) but in a different context.

[5] $U^i_y > 0$, $U^i_{yy} < 0$ for $y \in \left\{x^i, Z^i\right\}$.

[6] The questions of existence, uniqueness and stability of migration equilibria were investigated by Boadway and Flatters (1982), Hartwick (1980) and Stiglitz (1977). See also Appendix 5.3.

this result and Eq. (7), Eq. (9) can be rewritten as the following migration equilibrium condition:

$$U^1\left[\frac{F^1\left(N^1, \bar{L}^1\right) - S^{12} + S^{21} - T - Z^1}{N^1}, Z^1\right] + k\left(1 - N^1\right)$$
$$= U^2\left[\frac{F^2\left(1 - N^1, \bar{L}^2\right) - S^{21} + S^{12} + T - Z^2}{1 - N^1}, Z^2\right] + kN^1. \tag{10}$$

This relationship represents an implicit function $N^1 = N^1\left(S^{12}, S^{21}, T, Z^1, Z^2\right)$, if the derivative of (10) with respect to N^1 is not equal to zero. For stability of the migration equilibrium, we assume[7]:

$$D \equiv \frac{U_x^1}{N^1}\left(F_N^1 - x^1\right) + \frac{U_x^2}{N^2}\left(F_N^2 - x^2\right) - 2k < 0. \tag{11}$$

Differentiating Eq. (10) yields the following expressions:

$$\frac{\partial N^i}{\partial S^{ij}} = \frac{U_x^1/N^1 + U_x^2/N^2}{D} < 0, \tag{12}$$

$$\frac{\partial N^i}{\partial T} = \frac{\partial N^i}{\partial S^{ij}} < 0, \tag{13}$$

$$\frac{\partial N^1}{\partial Z^1} = \frac{U_x^1/N^1 - U_{Z^1}^1}{D}, \tag{14}$$

$$\frac{\partial N^1}{\partial Z^2} = -\frac{U_x^2/N^2 - U_{Z^2}^2}{D}. \tag{15}$$

These four migration response functions are needed in the following considerations.

From the point of view of a benevolent planner, the necessary first-order condition for the efficient provision of public goods is the usual Samuelson condition (for $i = 1, 2$):

$$N^i\frac{U_{Z^i}^i}{U_x^i} = 1. \tag{16}$$

The first-order condition for the efficient population distribution is as follows (Mansoorian and Myers 1993, p. 128):

$$-\frac{2kN^2}{U_x^2} \leq \left(F_N^1 - x^1\right) - \left(F_N^2 - x^2\right) \leq \frac{2kN^1}{U_x^1}. \tag{17}$$

Pareto-efficiency, i.e., a situation characterized by an efficient population distribution and an efficient provision of public goods, exists if both conditions (16) and (17) are simultaneously fulfilled.

[7] The stability problem has been investigated extensively by Stiglitz (1977).

3 Different timings of decisions

In this section, we will derive the equilibria resulting from different timings of the decisions of the governments within the framework of the model presented in Sect. 2 and examine their efficiency. The regional governments always set the transfers S^{ij} and the quantity of public goods Z^i autonomously, while the central government always determines the transfer T. This leads to the decisive difference to the models of Caplan et al. (2000), Köthenbürger (2007), Caplan and Silva (2011) and Duran-Vigneron (2012) in which the central government decides on the interregional transfer, whereas the regional governments only decide on the quantity of the public goods they provide. In the present model, by contrast, both levels of government can organize a transfer autonomously, such that the assignment problem can be analyzed.

3.1 Centralized leadership

In this section, we determine the equilibrium that arises given the following sequence of decisions:

Stage 1 The central government sets the interregional transfer T (vertical fiscal equalization), taking into consideration the anticipated response of the regional governments S^{ij} (horizontal fiscal equalization) and Z^i

Stage 2 The regional governments simultaneously decide on the horizontal transfers S^{12} and S^{21} and on the provision of public goods Z^1 and Z^2

The central government is thus the Stackelberg leader of a two-stage game, which is solved by backwards induction.

For each $\bar{T}$ specified by the central government, the regional governments, in Stage 2, choose Z^i and S^{ij} such that U^i is maximized in the respective region:

$$\max_{S^{ij},\, Z^i} U^i \left\{ \frac{F^i \left[N^i \left(S^{ij}, \bar{S}^{ji}, \bar{T}, Z^i, \bar{Z}^j \right), \bar{L}^i \right] - S^{ij} + \bar{S}^{ji} + (-1)^i \bar{T} - Z^i}{N^i \left(S^{ij}, \bar{S}^{ji}, \bar{T}, Z^i, \bar{Z}^j \right)}, Z^i \right\}. \tag{18}$$

Differentiating U^i w.r.t. S^{ij} yields:

$$\frac{dU^i}{dS^{ij}} = \frac{U_x^i}{N^i} \left[\frac{\partial N^i}{\partial S^{ij}} \left(F_N^i - x^i \right) - 1 \right]. \tag{19}$$

Taking (12) into account, we obtain the following Kuhn–Tucker condition for the optimal choice of S^{ij} in (18):

$$\left(F_N^i - x^i \right) - \left(F_N^j - x^j \right) + \frac{2kN^j}{U_x^j} \begin{array}{l} = 0, \quad \text{if } S^{ij} > 0, \\ \geq 0, \quad \text{if } S^{ij} = 0. \end{array} \tag{20}$$

All in all, we obtain:

$$-\frac{2kN^2}{U_x^2} \leq \left(F_N^1 - x^1\right) - \left(F_N^2 - x^2\right) \leq \frac{2kN^1}{U_x^1}. \tag{21}$$

Thus, condition (17) for an efficient population distribution is fulfilled in equilibrium for all given $\bar{T}$.

Following Wellisch (1994, p. 176), region 1 (2) is referred to as being transfer-constrained if the left (right) inequality in (21) strictly holds. This implies $S^{12} = 0 \left(S^{21} = 0\right)$ (Mansoorian and Myers 1997, p. 271). Only if there is equality in the left (right) inequality, transfer $S^{12} \left(S^{21}\right)$ can be nonzero. With imperfect mobility ($k > 0$), a case in which both horizontal transfers are nonzero, i.e., that neither region is transfer-constrained, is therefore ruled out.[8] In the following, only the case $S^{12} > 0$ and $S^{21} = 0$ will be considered.

Differentiating U^i with w.r.t. Z^i yields:

$$\frac{dU^i}{dZ^i} = U^i_{Z^i} + \frac{U_x^i}{N^i}\left[\frac{\partial N^i}{\partial Z^i}\left(F_N^i - x^i\right) - 1\right]. \tag{22}$$

Noting (14), we thus obtain the following first-order condition for the maximum of (18) w.r.t. $Z^i > 0$:

$$N^i \frac{U^i_{Z^i}}{U^i_x} = 1. \tag{23}$$

A comparison with efficiency condition (16) shows that the provision of the public good is efficient in both regions.

Therefore, the resulting Nash equilibrium—as described by (21) and (23)—is Pareto-efficient, if we neglect the possibility that the central government may intervene (Mansoorian and Myers 1993, p. 128).

In stage 1, the central government takes the above responses of the regional governments into account when setting transfer T. It optimizes a utilitarian welfare function for $\theta \in [0, 1]$ (Caplan et al. 2000, p. 271):

$$\begin{aligned}\max_T \Bigg\{ & \theta U^1\left[\frac{F^1\left[N^1(\cdot), \bar{L}^1\right] - S^{12}(T) + S^{21}(T) - T - Z^1(T)}{N^1\left[T, S^{12}(T), S^{21}(T), Z^1(T), Z^2(T)\right]}, Z^1(T)\right] \\ & + (1-\theta)\, U^2\left[\frac{F^2\left[1 - N^1(\cdot), \bar{L}^2\right] + S^{12}(T) - S^{21}(T) + T - Z^2(T)}{1 - N^1\left[T, S^{12}(T), S^{21}(T), Z^1(T), Z^2(T)\right]}, Z^2(T)\right]\Bigg\},\end{aligned} \tag{24}$$

where the symbol "·" in the numerators represents the arguments of N^1. These are written out in full in the denominators.

[8] This statement results from (21) with $U_x^i > 0$ and $N^i \neq 0$ for $i \in \{1, 2\}$.

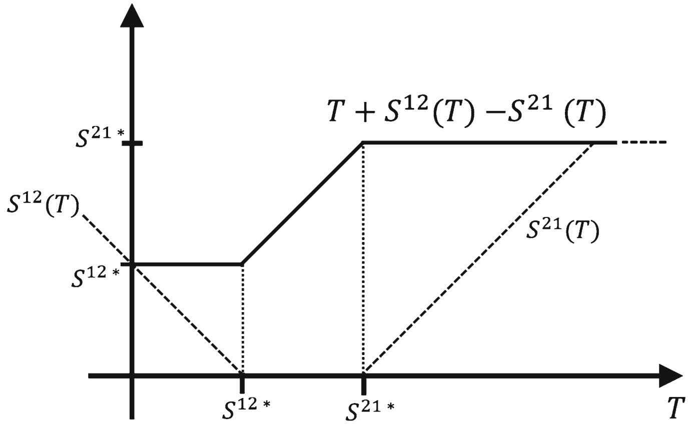

Fig. 1 Net transfer $T + S^{12}(T) - S^{21}(T)$ as a function of T

Noting (13), (14), (15), (23) and $S^{12} = 0$, the optimization of (24) provides the following first-order condition:

$$\theta \frac{U_x^1}{N^1}\left(1 + \frac{dS^{12}}{dT}\right)\left[\frac{\partial N^1}{\partial T}\left(F_N^1 - x^1\right) - 1\right] + (\theta - 1)\frac{U_x^2}{N^2}\left(1 + \frac{dS^{12}}{dT}\right)\left[\frac{\partial N^1}{\partial T}\left(F_N^2 - x^2\right) - 1\right] = 0. \tag{25}$$

Since in the optimization conditions for regional government 1 only the sum $S^{12}+T$ appears as an argument, the optimal reaction of this government in response to a change in T is in fact determined by the value of this sum. Two cases must be distinguished.

In the *first case*, region 1's government offers a positive transfer S^{12}. If the central government marginally changes its transfer T, the government of region 1 will seek to hold this sum at constant value to continue to achieve maximum utility. Thus, $\frac{dS^{12}}{dT} = -1$.[9] A rigorous proof for this relation is given in Appendix 5.1. Since, in this case, the central government cannot influence the optimization problems of the regional governments, the resulting subgame-perfect equilibrium is Pareto-efficient.

If, however, the central government chooses a transfer greater than the optimum transfer S^{12*} regional government 1 would be prepared to implement at $T = 0$, regional government 1 can no longer prevent a further increase in T by reducing S^{12}. This situation leads to the *second* case. As soon as $dS^{12}/dT = 0$ (*second* case), regional government 1 will make no transfer S^{12}, and the central government's optimum transfer fulfills $T^* > S^{12*}$. Both cases (as well as the case $S^{21} > 0$) are illustrated in Fig. 1, which shows the net interregional transfer $T + S^{12} - S^{21}$ as a function of T.

[9] This corresponds to the result obtained by Warr (1982) in a different context: "Donors respond to incremental fiscal redistributions by reducing their voluntary contributions by exactly a dollar for every dollar transferred in this way"(Warr 1982, p. 131).

As shown in Fig. 1, for $T = 0$, regional government 1 offers transfer S^{12*} that maximizes its utility function (18). Once the central government implements a nonzero transfer T, however, regional government 1 will reduce its transfer S^{12}, such that net transfer $T + S^{12} = S^{12*}$ and thus its utility remains the same ($dS^{12}/dT = -1$; first case as explained above). If the central government decided to implement a transfer of $T = S^{12*}$, regional government 1 would not implement any transfer S^{12} at all. This situation corresponds to the choice of $\theta = 1$ by the central government, i.e., if it solely maximizes the utility of the residents of region 1 (Mansoorian and Myers, 1997, 275, Lemma 1). If the central government selects a lower θ, transfer T increases, as $dT/d\theta < 0$ (see Appendix 5.2), but regional government 1 cannot counteract this increase anymore. It can only keep its transfer S^{12} to be zero ($dS^{12}/dT = 0$; second case as explained above). A lower θ and a corresponding higher transfer T therefore reduce the utility of regional government 1. For $\theta = 0$, the central government would finally consider solely the utility of the residents of region 2 and implement a transfer $T = S^{21*}$. The transfer realized for $\theta = 0$ thus also maximizes the utility of regional government 2. If the central government continued to increase its transfer T, the further migration to region 2 caused by such higher transfer would reduce the utility of regional government 2. To avoid such further migration, regional government 2 would counteract the increase in T by a correspondingly higher transfer S^{21} to regional government 1, such that the net transfer would remain constant: $T - S^{21} = S^{21*}$.

Therefore, the central government, by choosing T, can autonomously set the net transfer, but only between the values S^{12*} and S^{21*}. The precise value chosen by the central government will depend on the value $\theta \in [0, 1]$ selected for the social welfare function.

For the *second case*, i.e., for $dS^{12}/dT = 0$, $dS^{21}/dT = 0$, and a net transfer between the values S^{12*} and S^{21*}, we obtain from (25), after replacing $\partial N^1/\partial T$ by using Eq. (13):

$$\left(F_N^1 - x^1\right) - \left(F_N^2 - x^2\right) + \frac{2kN^2}{U_x^2}\left[\theta - (1-\theta)\frac{N^1}{N^2}\frac{U_x^2}{U_x^1}\right] = 0. \qquad (26)$$

Taking the extreme values $\theta = 1$ and $\theta = 0$, we obtain the following inequality:

$$-\frac{2kN^2}{U_x^2} \leq \left(F_N^1 - x^1\right) - \left(F_N^2 - x^2\right) \leq \frac{2kN^1}{U_x^1}. \qquad (27)$$

As a result, the central government chooses a transfer T such that inequality (27) holds, which is identical to efficiency condition (17). Since the regional governments realize for every given T the Samuelson condition (23) and hence efficiency condition (16), the subgame-perfect equilibrium is Pareto-efficient.

Proposition 1 *If the possibility of horizontal and vertical equalization exists, and the central government acts first, it is the central government alone that sets the size of the transfer, as long as the central government pursues distributional goals, i.e., as long as* $0 < \theta < 1$. *Then, horizontal equalization does not take place. The population* *distribution is efficient as well, and an efficient provision of the public goods results in both regions. Consequently, the subgame-perfect equilibrium is Pareto-efficient.*[10]

3.2 Decentralized leadership

In this section, we determine the equilibrium that arises when decisions are made in the following order:

Stage 1 The regional governments set the horizontal transfers S^{12} and S^{21} as well as the provision of the public goods Z^1 and Z^2, anticipating the response of the central government

Stage 2 The central government sets transfer T

The problem is again solved using backwards induction.

The central government sets transfer T for given values of $\bar{S}^{12}$, $\bar{S}^{21}$, $\bar{Z}^1$ and $\bar{Z}^2$ in stage 2. It chooses T such that social welfare W will be maximized:

$$\begin{aligned} W = \theta U^1 & \left\{ \frac{F^1\left[N^1\left(T, \bar{S}^{12}, \bar{S}^{21}, \bar{Z}^1, \bar{Z}^2\right), \bar{L}^1\right] - \bar{S}^{12} + \bar{S}^{21} - T - \bar{Z}^1}{N^1\left(T, \bar{S}^{12}, \bar{S}^{21}, \bar{Z}^1, \bar{Z}^2\right)}, \bar{Z}^1 \right\} \\ & + (1-\theta)\, U^2 \left\{ \frac{F^2\left[1 - N^1\left(T, \bar{S}^{12}, \bar{S}^{21}, \bar{Z}^1, \bar{Z}^2\right), \bar{L}^2\right] - \bar{S}^{21} + \bar{S}^{12} + T - \bar{Z}^2}{1 - N^1\left(T, \bar{S}^{12}, \bar{S}^{21}, \bar{Z}^1, \bar{Z}^2\right)}, \bar{Z}^2 \right\} \end{aligned} \tag{28}$$

The necessary first-order condition for a maximum is characterized by:

$$\theta \frac{U_x^1}{N^1}\left[\frac{dN^1}{dT}\left(F_N^1 - x^1\right) - 1\right] = (1-\theta)\frac{U_x^2}{N^2}\left[\frac{dN^1}{dT}\left(F_N^2 - x^2\right) - 1\right]. \tag{29}$$

The sufficient second-order conditions will be addressed in Appendix 5.3.

Inserting the derivative $\partial N^1/\partial T$ from (13) in (29) yields:

$$\left(F_N^1 - x^1\right) - \left(F_N^2 - x^2\right) + \frac{2kN^2}{U_x^2}\left[\theta - (1-\theta)\frac{N^1}{N^2}\frac{U_x^2}{U_x^1}\right] = 0. \tag{30}$$

This enables us to define the implicit function $T = T\left(S^{12}, S^{21}, Z^1, Z^2\right)$.

Taking the extreme values $\theta = 1$ und $\theta = 0$, we obtain the inequality:

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[10] "We can conclude that if the central government adopts a policy that implements the unitary state optimum, then any voluntary transfer will be completely crowded out. Of course, the above discussion is only suggestive since we have not analyzed when the various circumstances would occur. Further work is needed to characterize fully the relationship between the allocations under voluntary transfers and those of the unitary state optimum" (Boadway et al. 2003, p. 212).

$$-\frac{2kN^2}{U_x^2} \leq \left(F_N^1 - x^1\right) - \left(F_N^2 - x^2\right) \leq \frac{2kN^1}{U_x^1}, \tag{31}$$

which corresponds to efficiency condition (17). Therefore, we can again limit our considerations to the case $S^{12} > 0$ and $S^{21} = 0$ (i.e., that region 2 is transfer-constrained). Thus, we obtain $T = T\left(S^{12}, Z^1, Z^2\right)$.

Since the transfers only appear as a sum $-(S^{12} + T)$ or $S^{12} + T$ in the optimization condition (30), the central government will, regardless of the choice of S^{12}, set transfer T in such a way that this sum is preserved, i.e., $dT/dS^{12} = -1$, as long as $T < S^{12*}$. The proof can be found in Appendix 5.4. Since regional government 1 therefore cannot change the net transfer from the outset, carrying out a voluntary transfer is not meaningful, hence $S^{12} = 0$, and therefore $T = T\left(Z^1, Z^2\right)$.

So here too, it is solely the central government that sets transfer T within the bounds of inequality (31). The regional governments are left only with the task of optimizing their choice of Z^i. The central government has a second-mover advantage.

In stage 1, regional government i solves the following maximization problem:

$$\max_{Z^i} U^i \left\{ \frac{F^i\left\{N^i\left[T\left(Z^i, \bar{Z}^j\right), Z^i, \bar{Z}^j\right], \bar{L}^i\right\} + (-1)^i\, T\left(Z^i, \bar{Z}^j\right) - Z^i}{N^i\left[T\left(Z^i, \bar{Z}^j\right), Z^i, \bar{Z}^j\right]}, Z^i \right\}. \tag{32}$$

This yields the following first-order condition for the provision of Z^1:

$$\frac{dT}{dZ^1}\left[\left(F_N^1 - x^1\right) - \left(F_N^2 - x^2\right) + \frac{2kN^2}{U_x^2}\right] - \left(1 - N^1\frac{U_{Z^1}^1}{U_x^1}\right)\left[\left(F_N^2 - x^2\right) - \frac{2kN^2}{U_x^2}\right] = 0 \tag{33}$$

or

$$\frac{dT}{dZ^1}\left(\frac{F_N^1 - x^1}{\left(F_N^2 - x^2\right) - \frac{2kN^2}{U_x^2}} - 1\right) - 1 + N^1\frac{U_{Z^1}^1}{U_x^1} = 0. \tag{34}$$

An analogous condition exists for Z^2:

$$\frac{dT}{dZ^2}\left(\frac{F_N^2 - x^2}{\left(F_N^1 - x^1\right) - \frac{2kN^1}{U_x^1}} - 1\right) + 1 - N^2\frac{U_{Z^2}^2}{U_x^2} = 0. \tag{35}$$

The central government's response to a marginal change in the provision of the regional public goods, i.e., dT/dZ^1 and dT/dZ^2, can be obtained by implicitly differentiating (30) after inserting (7). Therein, it turns out that $dT/dZ^1 < 0$ and $dT/dZ^2 > 0$.

Therefore, an inefficiency arises, if one of the multipliers of dT/dZ^i in (34) or (35) is nonzero, i.e., if $\left(F_N^i - x^i\right) \neq \left[\left(F_N^j - x^j\right) - 2kN^j/U_x^j\right]$ $(i \neq j)$. In this case, (34) and (35) do not simplify to $N^i U_{Z^i}^i / U_x^i = 1$ $(i \in \{1, 2\})$, such that Samuelson condition (16) generally does not hold. In fact, none of the multipliers of dT/dZ^i in (34) or (35) vanishes, as shown below.

Since the central government chooses a transfer between S^{12*} and S^{21*}, both regional governments are transfer-constrained. Therefore, from (21), we have:

$$-\frac{2kN^2}{U_x^2} < \left(F_N^1 - x^1\right) - \left(F_N^2 - x^2\right) < \frac{2kN^1}{U_x^1}. \tag{36}$$

Thus, for region 1:

$$F_N^1 - x^1 > \left(F_N^2 - x^2\right) - \frac{2kN^2}{U_x^2},$$

and ($F_N^i - x^i < 0$, see Appendix 5.3) for the multiplier in (34):

$$\left(\frac{F_N^1 - x^1}{\left(F_N^2 - x^2\right) - \frac{2kN^2}{U_x^2}} - 1\right) < 0.$$

Similarly, for region 2:

$$F_N^2 - x^2 > \left(F_N^1 - x^1\right) - \frac{2kN^1}{U_x^1},$$

and for the multiplier in (35):

$$\left(\frac{F_N^2 - x^2}{\left(F_N^1 - x^1\right) - \frac{2kN^1}{U_x^1}} - 1\right) < 0.$$

The inefficient provision of the public goods Z^1 and Z^2 may be explained as follows: Regional government 1 may anticipate that the central government will, in Stage 2, choose T depending on the provision of Z^1. Since $dT/dZ^1 < 0$, regional government 1 will be inclined to provide an amount of Z^1 that is higher than the efficient one as this would be accompanied by a correspondingly lower value of T chosen by the central government, and thus a lower payment to be made by region 1. Similarly, regional government 2 would have an incentive to provide an amount of Z^2 that is higher than the efficient one, as such higher provision would increase the transfer T chosen by the central government ($dT/dZ^2 > 0$), and thus a higher payment to region 2.

Overall, the subgame-perfect equilibrium is not Pareto-efficient. This leads to

Proposition 2 *Also if the regional governments act first, there is only vertical equalization effected by the central government. Horizontal equalization does not take place. The resulting population distribution is efficient, but the provision of public goods is inefficient. The subgame-perfect equilibrium is therefore Pareto-inefficient.*

This latter inefficiency does not arise in Caplan et al. (2000) although they only replace both regional public goods with a federal public good whose provision is also determined by the two regional governments independently of each other. This marked difference can be explained as follows. In the model used here, each regional government decides autonomously on its provision of the public good. In Caplan et al., however, the two regional governments are linked to each other in the task of providing the federal public good via the reaction functions $Z^i = Z^i\left(Z^j\right), i \neq j$. The game in Caplan et al. thus has a structure corresponding to that of the Cournot duopoly model and should not be confused with the game played in the literature forming the basis of this paper.

3.3 Interim result

The results of Sects. 3.1 and 3.2 can be summarized as follows.

Regardless of the order in which the regional governments and the central government decide, in the framework of the model used here, the central government determines the redistribution across regions solely by means of its vertical equalization scheme. However, the subgame-perfect equilibrium is only Pareto-efficient, if the central government acts as a Stackelberg leader. The timing of the decisions thus plays a decisive role.

It is interesting to compare this result with the actual and the future fiscal equalization system of the Federal Republic of Germany. Today, in a first step, tax sharing arrangements are established between the two levels and between the Länder. After that, remaining disparate financial capacities of the Länder are reduced within the framework of a complex horizontal equalization scheme. Thereafter, in the framework of a vertical equalization scheme, the federal government provides grants to the Länder which are still financially weak.[11] Thus, the Federal Government, after all, decides on the extent of fiscal equalization in Germany. According to our model, having the regional governments act first, and only subsequently fixing vertical equalization does not lead to Pareto-efficiency. Interestingly, in the future (2020), the horizontal equalization scheme will be abolished in the Federal Republic of Germany, but vertical equalization will remain. As far as the findings in our model world may be transferred to this real-life change in fiscal equalization, they would indicate that abolishing horizontal equalization may not significantly affect the amount of redistribution across the regions, but possibly reduce an inefficiency in the provision of public goods. It may be an exciting new avenue for research in this field—which has largely remained in the model world realm—to empirically study corresponding effects upon the change of the equalization system in the Federal Republic of Germany.

[11] "Such law (a federal law, A&A) … may also provide for grants to be made by the Federation to financially weak Länder from its own funds to assist them in meeting their general financial needs (supplementary grants)." Article 107 (2) of the Basic Law for the Federal Republic of Germany.

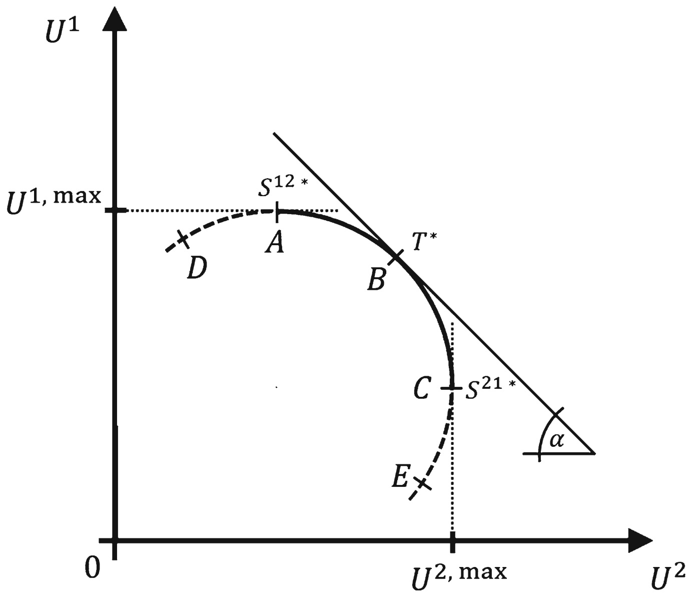

Fig. 2 Welfare maximum

4 Equal standards of living throughout the federation

"But the primary justification for fiscal equalization must be on equity grounds" (Oates 1999, p. 1128). The task of a fiscal equalization scheme across regions is not only to ensure an efficient distribution of population; it should, above all, also secure an equal standard of living throughout the federation.

It was shown in Sect. 3.1 that the government of region 1, when deciding on a horizontal transfer, will only have the utility of its own residents in mind. The transfer S^{12*} it wishes to effect, corresponds to the weight $\theta = 1$ in the social welfare function W (Mansoorian and Myers 1997, Lemma 1, p. 275). Region 2 is of interest to the government of region 1 only to the extent that it is available for admitting further residents from region 1 when it is overpopulated. The weight granted to the residents of region 2 by the government of region 1 is $\theta = 0$.

The central government, on the other hand, will assign positive weights to the residents of both regions to maximize welfare and therefore choose a transfer $T^* > S^{12*}$. A possible result of such a decision is shown in Fig. 2.

Movements along the dashed line from point D toward point A correspond to Pareto-improvements. The utility U^1 of region 1 and the utility U^2 of region 2 increase. For example, if regional government 1, starting from point D which is not a Pareto-optimum, increases its transfer S^{12}, its utility may be increased up to point A, as inhabitants of region 1 that consume more than they produce leave region 1, and the costs due to the increase of the transfer S^{12} are outweighed by the gain due to migration to region 2. At the same time, also the utility of regional government 2 increases in this step, since the transfer it receives for each immigrant from region 1 is higher than the costs caused by the immigration. If regional government 1 increased its transfer beyond point A, this would further increase the utility of region 2, but its own utility would be diminished. Therefore, point A represents a point on the utility-possibility frontier, from which Pareto-improvements are no longer possible.[12] Similar considerations apply, when considering point E as a starting point, from which Pareto-improvements may be achieved up to point C, by regional government 2 increasing its transfer S^{21}.

Overall, the utility-possibility frontier is indicated by the curve ABC, as exemplarily indicated in Fig. 2. The downward sloping tangent to the utility-possibility frontier at point B is an iso-welfare line with a gradient of

$$\frac{dU^1}{dU^2} = -\frac{1-\theta}{\theta}.$$

This welfare maximum can be achieved with a transfer T^* since a specific transfer can be assigned to each θ: $dT/d\theta < 0$ (see Appendix 5.2).

The possibility of influencing the utility distribution between the two regions by varying T disappears, however, when there is no attachment to home ($k = 0$) since then $dT/d\theta = 0$ (see Appendix 5.2). Since, in that case, region i maximizes $U^i\left(x^i, Z^i\right)$ subject to the constraint $U^i\left(x^i, Z^i\right) = U^j\left(x^j, Z^j\right)$, it is immediately evident that maximizing U^i maximizes U^j (Mansoorian and Myers 1993, p. 125). It is, therefore, not possible to influence the utility distribution by means of a transfer—and such a transfer is also not necessary since residents' utility has already been equalized through migration. The utility-possibility frontier dwindles to a point in this case.

Proposition 3 *If the population is perfectly mobile ($k = 0$), a vertical equalization scheme carried out for reasons of equality is neither possible nor necessary. If migration barriers exist ($0 < k < \infty$), the central government can ensure that the migration equilibrium is efficient by using a vertical equalization scheme, and it can at the same time pursue redistributive goals. If the barriers to migration are prohibitively high ($k \to \infty$), the central government can be guided solely by redistributive considerations in its choice of a vertical equalization scheme.*

5 Conclusions

In this paper, a federation consisting of two regions is considered, between which migration is possible. In each of the regions, a regional government decides on the quantity of the regional public good to be provided and a horizontal transfer to pay to the respective other region. A central government decides on a vertical transfer. The regional governments decide simultaneously on the quantity of the public goods to be provided. The two levels of government decide on horizontal and vertical fiscal equalization schemes in different chronological order.

The central finding here is that the central government will always prevail with the vertical equalization scheme irrespective of the timing of the decisions, if barriers

[12] See Atkinson and Stiglitz, 1980, 338.

to mobility are present. Horizontal equalization is neither effected nor necessary to attain constrained efficiency. The vertical equalization scheme leads to an efficient distribution of the entire population of the federation across both regions. The provision of the regional public goods, however, is only efficient, if the central government is the Stackelberg leader.

Appendix 5.1

Proof of $dS^{12}/dT = -1$ Inserting (7) in the first-order maximization conditions (21) and (23), and considering $S^{12} > 0$ as well as $S^{21} = 0$, one obtains:

$$\begin{aligned}&\left[F_N^1 - \frac{1}{N^1}\left(F^1 - S^{12} - T - Z^1\right)\right]\\&-\left[F_N^2 - \frac{1}{1-N^1}\left(F^2 + S^{12} + T - \bar{Z}^2\right)\right]\\&+\frac{2kN^2}{U_x^2\left[\frac{1}{1-N^1}\left(F^2 + S^{12} + T - \bar{Z}^2\right), \bar{Z}^2\right]} = 0,\end{aligned} \tag{37}$$

$$N^1\frac{U_{Z^1}^1\left[\frac{1}{N^1}\left(F^1 - S^{12} - T - Z^1\right), Z^1\right]}{U_x^1\left[\frac{1}{N^1}\left(F^1 - S^{12} - T - Z^1\right), Z^1\right]} = 1. \tag{38}$$

These are two implicit functions:

$$M^1\left(S^{12}, Z^1, T\right) = 0,$$

$$M^2\left(S^{12}, Z^1, T\right) = 0.$$

These can be explicitly solved if their Jacobian is nonzero:

$$S^{12} = S^{12*}(T)$$

$$Z^1 = Z^{1*}(T).$$

The rule for implicitly differentiating implicit functions leads to:

$$\begin{pmatrix}\partial M^1/\partial S^{12} & \partial M^1/\partial Z^1\\ \partial M^2/\partial S^{12} & \partial M^2/\partial Z^1\end{pmatrix}\times\begin{pmatrix}\partial S^{12*}/\partial T\\ \partial Z^{1*}/\partial T\end{pmatrix} = \begin{pmatrix}-\partial M^1/\partial T\\ -\partial M^2/\partial T\end{pmatrix}$$

and

$$\frac{dS^{12*}}{dT} = -\frac{\frac{\partial M^1}{\partial T} \times \frac{\partial M^2}{\partial Z^1} - \frac{\partial M^2}{\partial T} \times \frac{\partial M^1}{\partial Z^1}}{\frac{\partial M^1}{\partial S^{12}} \times \frac{\partial M^2}{\partial Z^1} - \frac{\partial M^2}{\partial S^{12}} \times \frac{\partial M^1}{\partial Z^1}}.$$

Since the Jacobian is nonzero, the function $S^{12} = S^{12*}(T)$ exists. Numerator and denominator are equal if

$$\partial M^1/\partial T = \partial M^1/\partial S^{12}, \quad \text{and} \quad \partial M^2/\partial T = \partial M^2/\partial S^{12}.$$

Since S^{12} and T are present in Eqs. (37) and (38) in an identical manner, this is the case. Thus:

$$\frac{S^{12*}}{dT} = -1.$$

Appendix 5.2

From (26) we obtain:

$$\left(F_N^1 - x^1\right) - \left(F_N^2 - x^2\right) + \frac{2kN^2}{U_x^2}\left[\theta - (1-\theta)\frac{N^1}{N^2}\frac{U_x^2}{U_x^1}\right] = 0.$$

With $x^1 = x^{11} - \frac{N^2 x^{12}}{N^1}$, $x^2 = x^{22} + x^{12}$ and $T = N^2 x^{12}$, this yields:

$$F_N^1 - \left(x^{11} - \frac{T}{N^1}\right) - \left[F_N^2 - \left(x^{22} + \frac{T}{N^2}\right)\right] + \frac{2kN^2}{U_x^2}\left[\theta - (1-\theta)\frac{N^1}{N^2}\frac{U_x^2}{U_x^1}\right] = 0$$

or, because $N^1 + N^2 = 1$,

$$\frac{T}{N^1 N^2} - \left(F_N^1 - x^{11}\right) - \left(F_N^2 - x^{22}\right) + \frac{2kN^2}{U_x^2}\left[\theta - (1-\theta)\frac{N^1}{N^2}\frac{U_x^2}{U_x^1}\right] = 0.$$

Implicit differentiation yields:

$$\frac{dT}{d\theta} = -2kN^1N^2\left(\frac{N^2}{U_x^2} + \frac{N^1}{U_x^1}\right) < 0.$$

For $k = 0$ we obtain

$$\frac{dT}{d\theta} = 0.$$

Appendix 5.3

The sufficient second-order condition for a maximum of (28) can be determined as follows (Boadway and Flatters 1982, 617, 617–619; Arnold 2005, for the case of regional public inputs). First, we will not consider transfers ($T = 0$). Thus:

$$x^1 = \frac{1}{N^1}\left[F^1\left(N^1, \bar{L}^1\right) - Z^1\right], \text{ and } x^2 = \frac{1}{N^2}\left[F^2\left(N^2, \bar{L}^2\right) - Z^2\right],$$

with $N^1 + N^2 = 1$. If the modified welfare function

$$\tilde{W} = \theta U^1\left\{\frac{F^1\left(N^1, \bar{L}^1\right) - Z^1}{N^1}, Z^1\right\} + (1-\theta)\, U^2\left\{\frac{F^2\left(1 - N^1, \bar{L}^2\right) - Z^2}{1 - N^1}, Z^2\right\} \tag{39}$$

is differentiated with respect to N^1, we obtain:

$$\frac{d\tilde{W}}{dN^1} = \theta U_x^1\left(\frac{F_N^1 - x^1}{N^1}\right) - (1-\theta)\, U_x^2\left(\frac{F_N^2 - x^2}{N^2}\right) \tag{40}$$

and

$$\begin{aligned}\frac{d^2\tilde{W}}{\left(dN^1\right)^2} = {} & \theta\left[U_{xx}^1\left(\frac{F_N^1 - x^1}{N^1}\right)^2 + \frac{U_x^1}{N^1}F_{NN}^1 - \frac{U_x^1}{N^1}\frac{2(F_N^1 - x^1)}{N^1}\right] \\ & + (1-\theta)\left[U_{xx}^2\left(\frac{F_N^2 - x^2}{N^2}\right)^2 + \frac{U_x^2}{N^2}F_{NN}^2 - \frac{U_x^2}{N^2}\frac{2(F_N^2 - x^2)}{N^2}\right].\end{aligned} \tag{41}$$

To determine the sign of (41), in a first step, we will look at the utility function of regional government 1, from the perspective of the central government ($Z^1 = \bar{Z}^1$):

$$\tilde{W}^1 = U^1\left[\frac{F^1\left(N^1, \bar{L}^1\right) - \bar{Z}^1}{N^1}, \bar{Z}^1\right]. \tag{42}$$

Differentiating with respect to N^1 yields:

$$\frac{d\tilde{W}^1}{dN^1} = U_x^1\frac{F_N^1 - x^1}{N^1}$$

and

$$\frac{d^2\tilde{W}^1}{\left(dN^1\right)^2} = U_{xx}^1\left(\frac{F_N^1 - x^1}{N^1}\right)^2 + \frac{U_x^1}{N^1}F_{NN}^1 - \frac{U_x^1}{N^1}\frac{2(F_N^1 - x^1)}{N^1}. \tag{43}$$

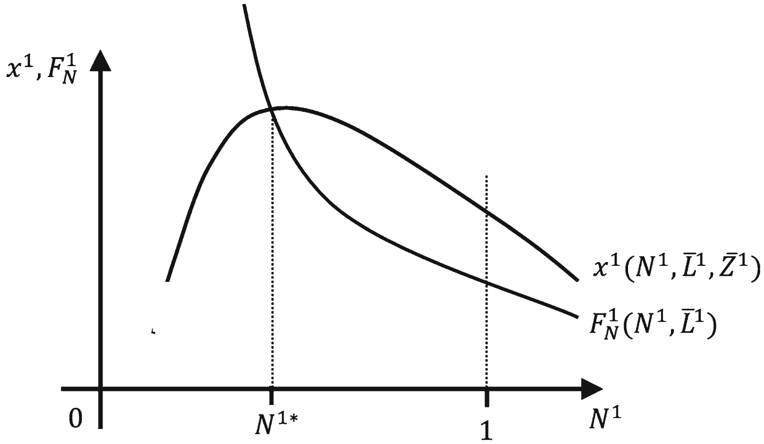

Fig. 3 Marginal productivity F_N^1 and *per capita* private consumption x^1

Hence, the first-order condition for a maximum at $N^1 = N^{1*}$ is:

$$F_N^1 - x^1 = 0. \tag{44}$$

Inserting this result into the second-order condition (43) yields:

$$\frac{d^2\tilde{W}^1}{\left(dN^1\right)^2} = \frac{U_x^1}{N^1} F_{NN}^1 < 0.$$

Therefore, the sufficient second-order condition for a maximum of $\tilde{W}^1\left(N^1, \bar{L}^1, \bar{Z}^1\right)$ is fulfilled in a vicinity of N^{1*}. On the left-hand side of N^{1*}, we have, within the mentioned vicinity:

$$\frac{d\tilde{W}^1}{dN^1} = U_x^1 \frac{F_N^1 - x^1}{N^1} > 0,$$

and thus $F_N^1 - x^1 > 0$. On the right-hand side of N^{1*}, we have:

$$\frac{d\tilde{W}^1}{dN^1} = U_x^1 \frac{F_N^1 - x^1}{N^1} < 0,$$

and thus $F_N^1 - x^1 < 0$.

In the following, we will assume that $d\tilde{W}^1/dN^1 > 0$ holds not only in a vicinity of N^{1*}, but in the entire interval $[0, N^{1*})$. Moreover, we will assume that $d\tilde{W}^1/dN^1 < 0$ holds in the entire interval $(N^{1*}, 1]$. In other words, we will assume that N^{1*} is not only a local but also a global maximum.

These assumptions are, from an economical point of view, reasonable, as illustrated in Fig. 3.

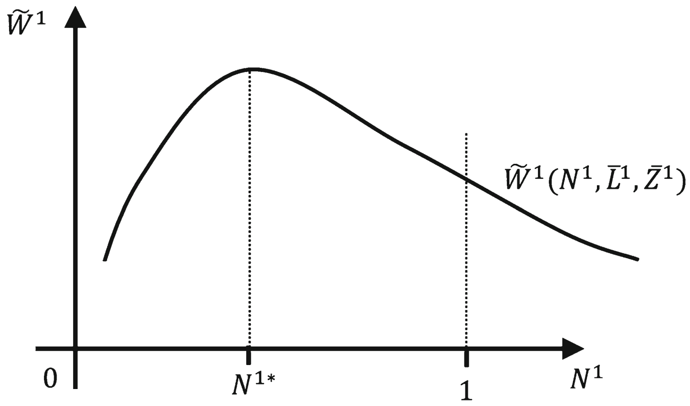

Fig. 4 $\tilde{W}^1$ as a function of N^1

Figure 3 shows the derivative F_N^1 of the production function (1). As we assumed a neoclassical production function, we have $F_N^1 > 0$ and $F_{NN}^1 < 0$. If we further assume that $F_{NNN}^1 > 0$ (which is the case for Cobb–Douglas functions, for example), F_N^1 is a convex and decreasing function of N^1, as shown in Fig. 3. Also shown in Fig. 3 is the per capita private consumption $x^1\left(N^1, \bar{L}^1, \bar{Z}^1\right)$. Since, at N^{1*}, $x_N^1 = F_N^1 - x^1 = 0$, x^1 and F_N^1 intersect at N^{1*}.

In the interval $[0, N^{1*})$, each immigrant's per capita consumption is lower than the respective immigrant's production: $F_N^1 > x^1$. Thus, each immigrant contributes to financing the public good. Thus, region 1 is underpopulated. At $N^1 = N^{1*}$, each immigrant's production equals his consumption: $F_N^1 = x^1$. Hence, the optimum population size is reached. In the interval $(N^{1*},1]$, increasing the population becomes disadvantageous for region 1, as each immigrant would consume more than he contributes to production. Without these advantages and disadvantages of an increased population, an economic theory of federalism would be obsolete.

It is noted that Fig. 3 assumes that x^1 has its maximum (as a function of N^1) in the interval $N^1 \in [0, 1]$, and thus F_N^1 and x^1 intersect in this interval. Of course, one may also look at the case, in which x^1 has its maximum at $N^1 > 1$ (or has no maximum at all). Then, the advantages of an increased population remain at all population sizes ($F_N^1 > x^1$), and the optimum population size would be $N^1 > 1$ (or $N^1 \to \infty$). In this case, the federation as a whole would be underpopulated. In the following, we will, however, assume that this is not the case, and instead assume that F_N^1 and x^1 intersect in the interval $N^1 \in (0, 1)$, and thus $x^1\left(N^1, \bar{L}^1, \bar{Z}^1\right)$ has a maximum within this interval.

Figure 4 correspondingly shows $\tilde{W}^1$ as a function of N^1.

Regarding Fig. 4, it should be noted that it may become convex starting from a specific value of N^1 (see also Boadway and Flatters 1982, 617–618). Notably, in the second-order condition (43), the first two terms on the right-hand side are negative. For $N^1 > N^{1*}$, however, we have $F_N^1 - x^1 < 0$. Therefore, the entire expression may possibly become positive, which would lead to $\tilde{W}^1$ becoming convex.

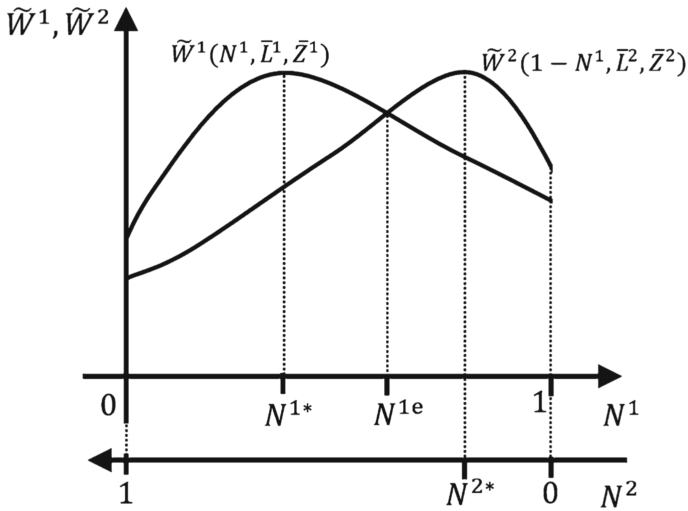

Fig. 5 $\tilde{W}^1$ and $\tilde{W}^2$ as a function of N^1 (or $N^2 = 1 - N^1$, respectively)

Similar considerations as outlined above for region 1 also apply to region 2 and its utility function $\tilde{W}^2$. If the utility functions $\tilde{W}^1$ and $\tilde{W}^2$ of both regions are shown in one graph, the following well-known Fig. 5 can be obtained (Flatters et al., 1974, 107; Stiglitz, 1977, 285; Hartwick, 1980, 696; Boadway and Flatters, 1982, 618).

The optimum populations N^{1*} and N^{2*} cannot be realized simultaneously, as $N^{1*} + N^{2*} < 1$, i.e., the federation is overpopulated. If no barriers to mobility exist, the migration equilibrium N^{1e} will be reached instead, in which both regions are overpopulated, such that $F_N^i - x^i < 0$.

Such a unique and stable equilibrium exists, however, only

- if the optimum size of each region has an "interior solution" (i.e., $N^{i*} \in (0, 1)$), and thus the advantages of increasing the population do not always apply (i.e., not in the entire interval of $N^1 \in (0, 1)$), and
- if both regions are overpopulated.

Then, we have $F_N^i - x^i < 0$, $i \in \{1, 2\}$, and the second-order condition for a maximum of (39), namely $d^2\tilde{W}/\left(dN^1\right)^2 < 0$ is met, if the functions $\tilde{W}^1$ and $\tilde{W}^2$ intersect at a position, at which both are concave and decreasing.

This migration equilibrium, however, is not Pareto-efficient without transfers. If transfers are allowed, the functions $\tilde{W}^1$ and $\tilde{W}^2$ are shifted upward and downward, respectively (if a net transfer occurs from region 2 to region 1). Thus, the utility of both regions may be increased (see Flatters et al., 1974, 107).

Appendix 5.4

Proof of $\frac{dT}{dS^{12}} = -1$ Inserting (7) into (30) yields:

$$\left[F_N^1 - \frac{1}{N^1}\left(F^1 - \bar{S}^{12} - T - \bar{Z}^1\right)\right] - \left[F_N^2 - \frac{1}{N^2}\left(F^2 + \bar{S}^{12} + T - \bar{Z}^2\right)\right]$$
$$+ \frac{2kN^2\theta}{U_x^2\left[\frac{1}{N^2}\left(F^2 + \bar{S}^{12} + T - \bar{Z}^2\right), \bar{Z}^2\right]}$$
$$- \frac{2kN^1(1-\theta)}{U_x^1\left[\frac{1}{N^1}\left(F^1 - \bar{S}^{12} - T - \bar{Z}^1\right), \bar{Z}^1\right]} = 0.$$

It can be seen immediately that T and S^{12} enter efficiency condition (30) in the same manner. Therefore, implicitly differentiating yields:

$$\frac{dT}{dS^{12}} = -1.$$

References

Anetsberger, G., & Arnold, V. (2017). Horizontal versus vertical fiscal equalization. *Discussion-paper Nr. 505,* Faculty of Economics, University of Hagen, Germany.

Arnold, V. (2005). Competitive versus cooperative federalism: Is a fiscal equalization scheme necessary from an allocative point of view? *Journal of Economics and Statistics, Jahrbücher für Nationalökonomie und Statistik, 225,* 259–282.

Atkinson, A. B., & Stiglitz, J. E. (1980). *Lectures on public economics* (1st ed.). New York: McGraw-Hill.

Boadway, R., Buettner, T., & Wildasin, D. E. (2008). New directions in fiscal federalism: Introduction. *Journal of Public Economics, 92,* 2285–2287.

Boadway, R., Cuff, K., & Marchand, M. (2003). Equalization and the decentralization of revenue-raising in a federation. *Journal of Public Economic Theory, 5,* 201–228.

Boadway, R., & Flatters, F. (1982). Efficiency and equalization payments in a federal system of governments: A synthesis and extension of recent results. *Canadian Journal Economics, 15,* 613–633.

Boadway, R., & Keen, M. (1996). Efficiency and the optimal direction of federal-state transfers. *International Tax and Public Finance, 3,* 137–155.

Buchanan, J. M., & Goetz, C. J. (1972). Efficiency limits of fiscal mobility: An assessment of the Tiebout model. *Journal of Public Economics, 1,* 25–43.

Caplan, A. J., Cornes, R. C., & Silva, E. R. D. (2000). Pure public goods and income redistribution in a federation with decentralized leadership and imperfect labor mobility. *Journal of Public Economics, 77,* 265–284.

Caplan, A. J., & Silva, E. C. D. (2011). Impure public goods, matching grand rates and income redistribution in a federation with decentralized leadership and imperfect labor mobility. *International Tax and Public Finance, 18,* 322–336.

Duran-Vigneron, P. (2012). When transparency goes against efficiency: Fiscal equalization in a context of firm and household mobility. *Journal of Public Economic Theory, 14,* 99–130.

Flatters, F., Henderson, V., & Mieszkowski, P. (1974). Public goods, efficiency, and regional fiscal equalization. *Journal of Public Economics, 2,* 99–112.

Hartwick, J. M. (1980). The Henry George rule, optimal population and interregional equity. *Canadian Journal of Economics, 13,* 695–700.

Köthenbürger, M. (2007). Ex-post redistribution in a federation: Implications for corrective policy. *Journal of Public Economics, 91,* 481–496.

Krelove, R. (1992). Efficient tax exporting. *Canadian Journal of Economics, 25,* 145–155.

Mansoorian, A., & Myers, G. M. (1993). Attachment to home and efficient purchases of population in a fiscal externality economy. *Journal of Public Economics, 52,* 117–132.

Mansoorian, A., & Myers, G. M. (1997). On the consequences of government objectives for economies with mobile populations. *Journal of Public Economics, 63,* 265–281.

Myers, G. M. (1990). Optimality, free mobility, and the regional authority in a federation. *Journal of Public Economics, 43,* 107–121.
Oates, W. E. (1999). An essay of fiscal federalism. *Journal of Economic Literature, 37,* 1120–1149.
Stiglitz, J. E. (1977). The theory of local public goods. In M. S. Feldstein & R. P. Inman (Eds.), *The economics of public services* (pp. 274–333). London: McMillan.
Warr, P. G. (1982). Pareto optimal redistribution and private charity. *Journal of Public Economics, 19,* 131–138.
Wellisch, D. (1994). Interregional spillovers in the presence of perfect and imperfect household mobility. *Journal of Public Economics, 55,* 167–184.