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The assignment and division of the tax base in a system of hierarchical governments

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Abstract Vertical externalities, changes in one level of government's policies that affect the budget of another level of government, may lead to non-optimal government policies. These externalities are associated with tax bases that are shared or "co-occupied" by two levels of government. Here I consider whether co-occupancy of tax bases is desirable. I examine the optimal extent of the tax bases of a lower level of government (local) and a higher level (state). I find that it is optimal to have co-occupancy in the absence of other corrective policies if commodities in the tax bases are substitutes. Further, if the state government can differentially tax the co-occupied segment of the tax base and the segment it alone taxes it will obtain the (second-best) outcome obtained with other policy instruments such as intergovernmental grants.

Keywords Fiscal competition · Vertical externalities · Tax base co-occupancy

JEL Classification H20 · H71 · H73 · R12 · R28 · R41

1 Introduction

While the concept of a horizontal fiscal externality arising from tax competition among governments at the same level has been the topic of numerous papers for more than thirty years, the focus on "vertical" fiscal externalities received later attention.

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As the name vertical implies, these externalities arise between governments at different levels, for example between state and local governments or federal and state governments. In this case the focus is on the overlap in the tax bases of two levels of government. An example from Dahlby (1996) is the excise tax placed on cigarettes by both the federal and provincial governments in Canada. When choosing its tax rate, each province presumably only considers the tax's impact on its own revenues and ignores the impact on the federal government. As a result of an increase in the province's tax rate federal tax revenues will decrease because of the reduction in cigarette purchases, a vertical externality. Because of the impact on the revenue of the federal government, the cost of funds perceived by the province differs from the social cost of the funds. Then the externality is negative as increases in the state tax reduce federal revenues. Because the state government ignores this negative externality, it will overtax cigarettes.[1]

Vertical fiscal externalities act in both directions—federal taxes will also affect state revenues. Flowers (1988), Wrede (1996), and Keen and Kotsogiannis (2002), for example, assume that both levels of government ignore the vertical externality imposed on the other level of government when setting tax policies, leading to excessive taxation at both levels of government. Empirical studies estimating tax reactions among levels of governments (for example, Besley and Rosen 1998; Devereux et al. 2007; Fredriksson and Mamun 2008; Brülhart and Jametti 2006) generally find that vertical externalities have significant impacts on tax policies in federal systems.

While early studies focused on vertical externalities in a single market, research has extended to consider the impacts of co-occupancy in a multiple market including studies by Dahlby (1996, 2008), Keen (1998), Hoyt (2001), Dahlby et al. (2000), Dahlby (2001), and Dahlby and Wilson (2003). In addition to having multiple tax bases, different levels of governments rely on very different sources of revenue. As Table 1 suggests, there is only limited overlap or co-occupancy in sources of revenue of state and local governments in the USA. As can be seen from the table, while local governments collect over 30% of their tax revenue from the property tax, state governments only collect 0.8%; conversely, state governments collect 27% of their tax revenue from the personal income tax, 15.6% from a general sales tax, and 29.6% from social insurance/retirement taxes (payroll), while the shares for local government revenues are, respectively, 3.1, 5.5, and 7.4%. Despite the limited co-occupancy there is likely to be a strong link between their alternative tax bases. Changes in a major source of state revenue such as the personal income tax will undoubtedly affect revenues from the property tax, a major source of local revenue. Thus, while vertical fiscal externalities will almost certainly arise in a co-occupied tax base, it does not follow that eliminating co-occupancy eliminates fiscal externalities, an idea that may underlie the recommendation by some to eliminate co-occupancy.

The issue addressed here, what level of government should tax what goods or services or inputs is referred to in the federalism literature as the "assignment" problem. In a surprisingly small literature, the best known discussion is Musgrave (1983). Dahlby

[1] To the extent that there is cross-border shopping and tax-induced smuggling of cigarettes, there is also a "horizontal" fiscal externality as the increase in the sales tax rate in one province will presumably increase tax revenues from cigarettes sales in other provinces.

Table 1 Percent of total own source by tax base, state, local, and federal, 2012[1]

Tax base	Local	State	Federal
Property	30.9	0.8	N/A
General sales	5.5	15.6	N/A
Excise/selective sales	3.1	11.9	3.4
Individual income	3.1	26.7	44.0
Corporate income	0.8	3.9	9.1
Motor vehicle/other	2.8	7.4	6.4
Social insurance/retirement	7.4	29.6	37.1

1. From *Government Finances Statistics* (http://www.census.gov/govs/financegen/index.html)

(2001) provides a summary of a "consensus" among economists that, as Dahlby notes, follows Musgrave (1983) recommendations regarding the assignment of tax bases between central (federal), middle (state/province), and lower (local) governments.[2] Paraphrasing Dahlby (2001), lower-level jurisdictions should tax bases with less inter-jurisdictional mobility and that are cyclically stable, while higher-level (central) ones should undertake progressive, personal taxes, other taxes suitable for purposes of stabilization policy, and tax bases that are distributed unequally among jurisdictions. Benefit taxes are appropriate at all levels of governments. As Dahlby (2001), p. 94–95 notes, what is not addressed in this list of rules for assignment is the issue of fiscal externalities and its implications for co-occupancy. While Dahlby offers some informal guidelines, there is no formal discussion of the whether, where, and to what extent co-occupancy of tax bases is advisable.

Numerous studies have considered policies by the higher level of government to correct for the vertical externalities created by taxes imposed by the lower level of government. Corrective policies include separating the tax bases of the two levels of government (Flowers 1988); increasing the number of lower-level governments (Keen 1995; Keen and Kotsogiannis 2004); and providing intergovernmental grants (Dahlby 1996; Boadway and Keen 1996; Boadway et al. 1998; Flochel and Madies 2002). However, consideration of how to allocate the tax base among the levels of government has been quite limited. Keen (1998) does devote some discussion (and analysis) to co-occupancy and assignment by addressing the question of whether it is better to co-occupy an inelastic tax base or a more elastic tax base. In different contexts Kotsogiannis (2010), Haufler and Lulfesmann (2015), and Kotsogiannis and Raimondos (2015) consider "optimal" co-occupancy. However, in Kotsogiannis (2010) the focus is on the use of equalization grants to correct for vertical fiscal externalities *given* co-occupied tax bases, while in both Haufler and Lulfesmann (2015) and Kotsogiannis and Raimondos (2015) the co-occupancy corrects for horizontal externalities: in the case of Haufler and Lulfesmann (2015) these are associated with capital taxation by asymmetric countries and in the case of Kotsogiannis and Raimondos (2015) countries levy taxes to change the terms of trade.

Here I address the assignment question using a very different framework from those in either Musgrave (1983) or Keen (1998). Following Dahlby (1996), Hoyt (2001) and

[2] Other summaries and discussions of Musgrave (1983) include Musgrave and Musgrave (1989), Oates (1994), and Keen (1998).

Keen and Kotsogiannis (2004), I assume that both levels of government maximize the welfare of their residents rather than acting as "Leviathan" maximizing government revenue (for example, Flowers 1988; Wrede 1996, 2000; Keen and Kotsogiannis 2003). However, while assuming the lower-level government ignores the impacts of its tax policies on the higher-level government (state) I consider both the possibility that the state government ignores or considers the impacts of its tax policies on local revenues.

Rather than considering the type of tax base that should be taxed by different levels of government, I first consider how to divide a uniform tax base among two levels of government and whether co-occupancy is desirable or not. While admittedly an abstract and simplistic model that ignores the Tiebout considerations found in, for example, Brueckner (2000) and Brueckner (2004) as well as any of the issues associated with tax assignment such as geography, benefit taxation, and cross-border shopping discussed in Bird (2000) and McLure (2001), it enables me to address the question of whether the existence of vertical fiscal externalities might, as suggested by Flowers (1988) and Dahlby (2001) among others, lead to the conclusion that there should be no or very limited co-occupancy among tax bases.

Specifically, by highlighting the interdependence across tax bases generated by nonzero cross-price elasticities among commodities in the tax base, I demonstrate, like Dahlby (2008), that elimination of co-occupancy will not eliminate vertical fiscal externalities. As a consequence, even in the absence of co-occupancy the tax rates of the two levels of government will not be optimally set. If the commodities in the tax base are gross substitutes, eliminating co-occupancy results in a positive fiscal externality, meaning that tax rates will become "too" low. Because both the tax rates and tax bases of governments are policy instruments as the extent and direction of the fiscal externalities associated with tax increase and those associated with increases in tax bases can be quite different. In fact, I find that in the case in which commodities are gross substitutes, co-occupancy, at least to some extent, is optimal; when commodities are gross complements, it is unlikely that co-occupancy is optimal.

The basic model, found in Sect. 2, is of a continuum of commodities with identical demands along the lines, for example, of Dixit and Stiglitz (1977), Yitzhaki (1979), and Wilson (1989). In this section, I consider the equilibrium tax rates and bases when the two levels of government (state and local) choose them independently. In Sect. 3 I first describe the social-welfare-maximizing division of the tax base between the two levels of government in the absence of any overlap. I then consider whether, and under what conditions, would co-occupancy be socially optimal. As well, I allow for the possibility that the state government can set different tax rates on the base that it alone taxes and the base that it shares (co-occupies) with the local government. Section 4 concludes.

2 Tax choices with independent governments

2.1 A simple model

I consider an economy with a single state government and n local governments with each locality having a single, identical resident. Each government provides a public service to its residents with g_s being the level provided by the state government and $g_j,\ j = 1, \ldots, n$ the level provided by locality *j*. Both public services are produced with constant costs with the cost of providing g_s to the n localities equal to ng_s and the cost of providing the local public service in locality *j* equal to g_j. That the cost function for the state public service is ng_s is not intended to suggest that the costs of the state public service are a function of the population or number of localities but, instead, allows for easy comparison of the provision of the two public services.[3] While there are n independent localities, as each local government has the same policy objectives, in equilibrium all localities choose the same policies. Then given this symmetry, I denote local policies by the subscript *l*. To further simplify the analysis, I also assume that the number of localities is large enough so that no individual locality considers the impacts its policies have on state revenues.

In addition to the public services, residents also consume private commodities. Following Dixit and Stiglitz (1977), Yitzhaki (1979), and Wilson (1989), I consider a continuum of these private commodities identified on the interval [0,K]. While the interval of commodities is [0,K] only the interval [0,1] is subject to taxation by either the state or local governments.[4] As my interest is in how to allocate the tax base between the two levels of government, like Dixit and Stiglitz (1977) I assume identical demand functions over the set of commodities. By this I mean that when the prices of two commodities are identical, the quantity demanded is the same for both. The utility function can be represented by

$$U = \int_0^K \left(U^x(x(q(k), Q)\right) \mathrm{d}k + U^l\,(g_l) + U^s\,(g_s) \tag{2.1}$$

where g_l and g_s refer to the local and state public services. The gross of tax price of commodity k, $x(k)$, is denoted by $q(k)$ with the net of tax prices for all commodities equal to unity. The term $Q = \left(\int_0^K q(k)\mathrm{d}k\right)$ is an index of all commodity prices. It should be noted that the separable utility function in public goods means that in the second-best (central government) policy the marginal rates of substitution for the two

[3] Given this simple framework, it might be asked why there are two levels of government. While I have the cost function for the state public service as a function of population, the idea is that this is a service that is provided uniformly throughout the state without the possibility of varying services for different localities and with costs prohibitively high for localities to provide themselves. Then following Oates' (1972) proposition that the government providing the public service should be the one that incorporates the extent of the benefits received from the service, here the state is the appropriate level. It, too, might be argued that there is not a need for local governments as they are all identical. This is a simplification—allowing for heterogeneity in public service preferences and therefore taxes would qualitatively change some results but not the basic premises.

[4] While we may think of the commodity space as a straight line or circle, like Dixit and Stiglitz (1977) the distance between any two commodities has no bearing on the relationship between them, that is, the degree to which they are substitutes or complements. This is seen in the formulation of the price index.

public goods are equal. If the public goods did not enter into utility separately this might not be the case. Having the marginal rates of substitution equal in the second-best solution enables us to better understand and compare the policies when governments choose policies independently or face limitations on their tax bases to the second-best policies. Unlike Dixit and Stiglitz (1977), I assume a relatively general form of the utility function. As the demand functions for commodities are identical then when the prices of commodities are the same so are their demands, for my purposes, an important implication of having identical commodities is that the optimal tax structure is extremely simple—all commodities are taxed equally.[5]

As both the local and state governments assess uniform commodity taxes to finance their public services, the gross price of each commodity depends on whether it is part of the tax base for the local, state, or both governments. Localities tax the set of commodities on the interval $\left[0, \bar{K}_l\right]$, while the set taxed by the state government is on the interval $\left[\overline{K}_s, 1\right]$. Let $k_l = \min\left(\overline{K}_l, \overline{K}_s\right)$, $k_s = 1 - \max\left(\overline{K}_l, \overline{K}_s\right)$ and $k_{ls} = \max(\overline{K}_l - \overline{K}_s, 0)$ denote, respectively, the length of interval taxed only by the local government, only by the state government, and by both levels of government. As both governments have the incentive to tax any untaxed base, co-occupancy occurs ($k_{ls} > 0$) when $k_l + k_s < 1$. Then the gross of tax price for the commodities can be summarized by

$$q(k) = \begin{cases} 1+\tau_l, & k \in 0, \left[\min\left(\overline{K}_l, \overline{K}_s\right)\right] \\ 1+\tau_s, & k \in \left[\max\left(\overline{K}_l, \overline{K}_s\right), 1\right] \\ 1+\tau_l+\tau_s, & k \in \left[\overline{K}_s, \overline{K}_l\right] \; if\,,\; \overline{K}_l > \overline{K}_s \end{cases} \tag{2.2}$$

where τ_l and τ_s denote the local and state tax rates, respectively.[6]

Figure 1 illustrates the division of the tax base with no overlap (no co-occupancy) and with overlap (co-occupancy).

[5] Further assumptions regarding the elasticity of the demands of the products are necessary for this result as will be seen later. Alternatively, that there are uniform tax rates across commodities can also be considered an assumption reflecting (most) state and local sales tax in the USA and VAT systems elsewhere as discussed in Wilson (1989).

[6] With these taxes the price index is

$$Q = \begin{cases} \int_0^{\overline{K}_s} (1+\tau_l)\,\mathrm{d}k + \int_{\overline{K}_s}^{\overline{K}_l} (1+\tau_l+\tau_s)\,\mathrm{d}k + \int_{\overline{K}_l}^{1} (1+\tau_s)\,\mathrm{d}k + \int_1^K \mathrm{d}k & \\ \quad = K + k_l\tau_l + k_s\tau_s + k_{ls}(\tau_l+\tau_s) & \overline{K}_l > \overline{K}_s \\ \int_0^{\overline{K}_l} (1+\tau_l)\,\mathrm{d}k + \int_{\overline{K}_l}^{\overline{K}_s} \mathrm{d}k + \int_{\overline{K}_s}^{1} (1+\tau_s)\,\mathrm{d}k + \int_1^K \mathrm{d}k & . \\ \quad = K + k_l\tau_l + k_s\tau_s & \overline{K}_l < \overline{K}_s \end{cases}$$

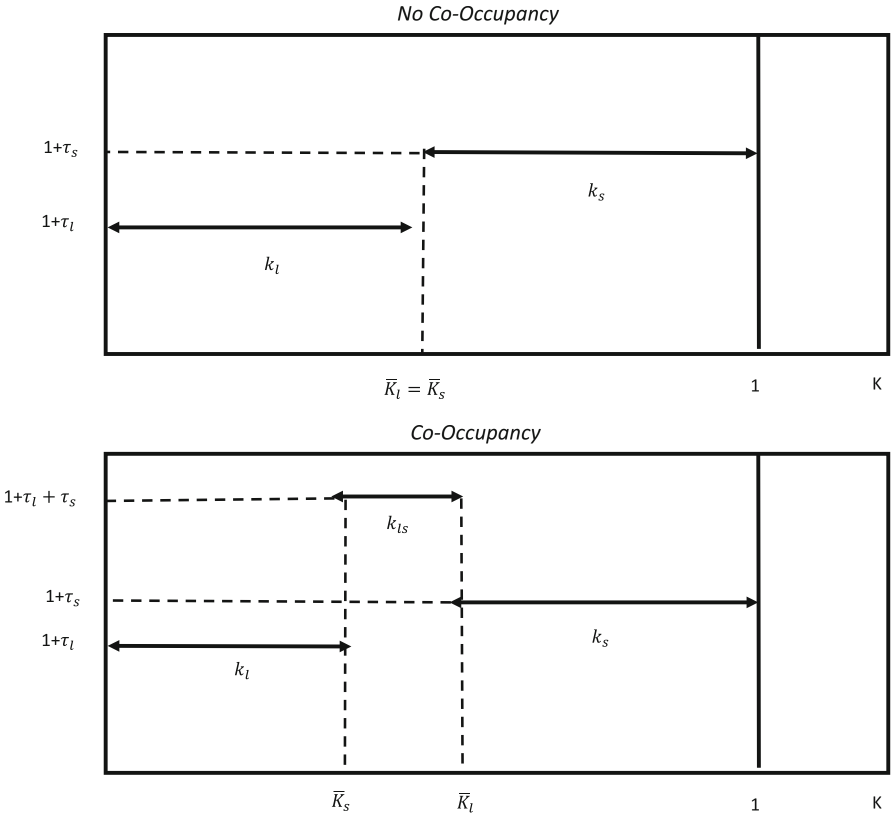

Fig. 1 The tax bases

Then the indirect utility function can be expressed as

$$V\left[\tau_l, \tau_s, \overline{K}_s, \overline{K}_l\right] = \int_0^K V\left(q(k), Q\right) \mathrm{d}k + U^s\left(g_s\left(\tau_l, \tau_k, \overline{K}_s, \overline{K}_l\right)\right) + U^l\left(g_l\left(\tau_l, \tau_k, \overline{K}_s, \overline{K}_l\right)\right) \tag{2.3}$$

where

$$\int_0^1 V\left(q(k), Q\right) \mathrm{d}k = \begin{cases} \int_0^{\overline{K}_l} U^x\left(x(q(k) = 1+\tau_l, Q)\right) \mathrm{d}k + \int_{\overline{K}_l}^{\overline{K}_s} U^x\left(x(q(k) = 1, Q)\right) \mathrm{d}k \\ + \int_{\overline{K}_s}^{1} U^x\left(x(q(k) = 1+\tau_s, Q)\right) \mathrm{d}k + \int_1^K U^x\left(x(q(k) = 1, Q)\right) \mathrm{d}k, \ \overline{K}_l < \overline{K}_s \\ \int_0^{\overline{K}_s} U^x\left(x(q(k) = 1+\tau_l, Q)\right) \mathrm{d}k + \int_{\overline{K}_l}^{\overline{K}_s} U^x\left(x(q(k) = 1+\tau_l+\tau_s, Q)\right) \mathrm{d}k \\ + \int_{\overline{K}_s}^{1} U^x\left(x(q(k) = 1+\tau_s, Q)\right) \mathrm{d}k + \int_1^K U^x\left(x(q(k) = 1, Q)\right) \mathrm{d}k \end{cases}, \ \overline{K}_l > \overline{K}_s \tag{2.4}$$

2.1.1 Government objectives

The objective function for local governments is given by

$$W^l\left[\tau_l, \tau_s, \overline{K}_l, \overline{K}_s\right] = \int_0^K V(q(k), Q)\mathrm{d}k + U^l\left(g_l\left(\tau_l, \tau_s, \overline{K}_l, \overline{K}_s\right)\right) \quad (2.5)$$

where the level of the local public service is implicitly defined by the state and local tax rates and bases, $g_l\left(\tau_l, \tau_s, \overline{K}_l, \overline{K}_s\right)$. As mentioned, the large number of localities means any impact of their individual policies on state revenues is small and therefore ignored by them.

For the state government consider the objective,

$$\begin{aligned} W^s\left[\tau_l, \tau_s, \overline{K}_l, \overline{K}_s\right] = &\int_0^K V(q(k), Q)\mathrm{d}k + U^s(g_s(\tau_l, \tau_s, \overline{K}_s, \overline{K}_l)) \\ &+\alpha U^l(g_l(\tau_l, \tau_s, \overline{K}_s, \overline{K}_l)) \end{aligned} \quad (2.6)$$

where $\alpha \in [0, 1]$. The parameter α denotes the extent that the state government considers the impacts of its policies on the local government revenues. If $\alpha = 1$ the state government fully considers the impacts of its policies on local revenues and the welfare of its residents; at the other extreme ($\alpha = 0$) the state ignores the impacts of its policies on local revenues and public services. One explanation for the alternative weights on $U^l(g_l)$ could be resident/voter knowledge of the impacts that state tax policies have on local revenues. If voters are fully informed about these impacts, then in their own self-interest elected public officials would consider them; if residents are unaware of the impacts of state taxes on local revenues and only evaluate state policy based on its impacts on prices and the level of the state public good it may be in the interest of elected official to ignore impacts on local revenues.

2.1.2 Government budget constraints

The state budget constraint is given by $ng_s = n\tau_s\left(k_s x_s + k_{ls}x_{ls}\right) = n\tau_s X_s$, and the local budget constraint is given by $g_l = \tau_l\left(k_l x_l + k_{ls}x_{ls}\right) = \tau_l X_l$ where x_s, x_l, and x_{ls} denote the demand for commodities subject to the state tax only, to the local tax only, and to both taxes, respectively.

Critical to understanding the tax rates chosen by the two levels of government and the optimal tax bases for them is understanding the impacts of changes in their tax rates and bases on their revenues. These impacts, summarized below, are derived in Supplementary Appendix A.1.

$$\frac{\mathrm{d}g_j}{\mathrm{d}\tau_j} = X_j\left[1 + \tau_j\left(\epsilon_{11} + \left(k_j + k_{ls}\right)\epsilon_{21}\right)\right] \text{ and} \quad (2.7a)$$

$$\frac{\mathrm{d}g_i}{\mathrm{d}\tau_j} = \tau_i X_i\left[\frac{k_{ls}}{(k_{ls} + k_i)}\frac{x_{ls}}{\overline{x}_i}\epsilon_{11} + (k_s + k_{ls})\epsilon_{21}\right], \ i, j = l, s; i \neq j \quad (2.7b)$$

where $\overline{x}_i = \frac{k_{ls}x_{ls}+k_i x_i}{k_{ls}+k_i}$, $i = l, s$. In (2.7a) and (2.7b) I denote the own-price percentage change in demand by $\epsilon_{11} \equiv \frac{\partial x(k)}{\partial q(k)} \frac{1}{x(k)}$ and the cross-price percentage change (with respect to the index Q) by $\epsilon_{21} \equiv \frac{\partial x(k)}{\partial Q} \frac{1}{x(k)}$ where, at the pre-tax prices, $\epsilon_{21} = -\frac{1}{K}(1+\epsilon_{11})$. For simplification, ϵ_{11} and ϵ_{21} are treated as constant in the analysis and the same regardless of whether the commodity is in the state-only, local-only, or shared tax base. Alternatively, they can be interpreted as the weighted average of the different partitions of the tax base.[7] Then from (2.7a), it can be seen that the impact of a tax increase on own-tax revenues depends on the size of the tax base (X_j), the tax rate, own-price effect (ϵ_{11}), and the cross-price effects of commodities within the tax base, $\left(k_j + k_{ls}\right)\epsilon_{21}$.

The sign and magnitude of the fiscal externality, the impact of an increase in a tax rate by one level of government on the tax revenues of the other level, (2.7b), depends on both the product of the overlap in the tax bases and own-price effect $\left(\frac{k_{ls}}{(k_{ls}+k_i)}\frac{x_{ls}}{\overline{x}_i}\epsilon_{11}\right)$ and the impact of the cross-price effects on the tax base $\left(\left(k_j + k_{ls}\right)\epsilon_{21}\right)$. If $\epsilon_{21} < 0$, that is, the goods in the tax base are complements, $\frac{\mathrm{d}g_i}{\mathrm{d}\tau_j}$ will be negative; if goods in the tax base are substitutes ($\epsilon_{21} > 0$) then the sign of $\frac{\mathrm{d}g_i}{\mathrm{d}\tau_j}$ depends on the extent of the overlap in the two tax bases and the ratio of the cross-price and own-price effects, $\frac{-\epsilon_{21}}{\epsilon_{11}}$.

While the focus of the literature on fiscal externalities has been on tax rates, I am also interested in the impacts of changes in tax bases of the two levels of government. Then the impacts of increases in the tax base on revenues are

$$\frac{\mathrm{d}g_j}{\mathrm{d}\overline{K}_j} = \tau_j\left[x_z + \tau_l\left(k_{ls}+k_j\right)X_j\epsilon_{21}\right], i, j = l, s \text{ and} \tag{2.8a}$$

$$\frac{\mathrm{d}g_i}{\mathrm{d}\overline{K}_j} = \tau_j\tau_i\left[Dx_z\epsilon_{11} + \left(k_{ls}+k_j\right)X_i\epsilon_{21}\right],\ i, j = l, s; i \neq j \tag{2.8b}$$

where $D = 0\,(1)$ and $z = j\,(ls)\,if\ \overline{K}_l\ < (>)\,\overline{K}_s,\ \ j = l, s$.[8] As is the case with tax rates, changes in the tax base of one level of government affect tax revenues of the other level of government. The impact on own revenues of an expansion of tax base is: (1) the direct effect of adding an additional commodity $\left(\tau_j x_z,\ z = j\,(ls)\right)$ and (2) the effect the increase in the price of the added commodity (τ_j) has on the demand for commodities in its existing base $\left(\tau_l\left(k_{ls}+k_j\right)\epsilon_{21}\right)\tau_j X_j$ that depends on

[7] For example, without the assumption of constant and equal percentage changes, in (2.7a) we have $\frac{dg_l}{d\tau_l} = \left[x_{ls} + x_l + \tau_l\left(\epsilon^l_{11}x_l + \epsilon^{ls}_{11}x_{ls}\right) + (k_{ls}+k_l)\left(\epsilon^l_{21}x_l + \epsilon^{ls}_{21}x_{ls}\right)\right]$ where the superscript refers to whether it is the segment with only the local tax (l) or subject to both taxes (ls). Then we can think of $\epsilon_{11}\,(x_{ls}+x_l) = \epsilon^l_{11}x_l + \epsilon^{ls}_{11}x_{ls}$ and $\epsilon_{21}\,(k_{ls}+k_l)\,(x_{ls}+x_l) = (k_{ls}+k_l)\left(\epsilon^l_{21}x_l + \epsilon^{ls}_{21}x_{ls}\right)$. While ϵ_{11} and ϵ_{21} are not elasticities they are related, differing only by the exclusion of prices. In the numerical example, I assume constant elasticities and allow the percentage changes to vary with price.

[8] An increase in the state tax base means a decrease in $\overline{K}_s$ so rather than $\frac{\mathrm{d}g_s}{\mathrm{d}\overline{K}_s}$ and $\frac{\mathrm{d}g_l}{\mathrm{d}\overline{K}_s}$ (2.8a) is for $-\frac{\mathrm{d}g_s}{\mathrm{d}\overline{K}_s}$ and $-\frac{\mathrm{d}g_l}{\mathrm{d}\overline{K}_s}$.

whether commodities are substitutes or complements. The impact of the expansion of the one level's tax base on the other level's tax revenue depends on whether the two tax bases overlap and the cross-price elasticities (effects) among commodities—when the two tax bases completely overlap, increases in the local (state) tax base will always decrease the state (local) tax base.

2.2 Fiscal externalities and tax rates

If a single, centralized government were to finance both public goods, given the assumption of uniform demands it would apply a uniform tax rate on all commodities and allocate revenue to ensure the marginal rate of substitution (MRS) for the two public goods are equal. When the two governments independently tax and finance their public services, the co-occupancy of tax bases generates negative fiscal externalities and, as a result, may lead to over-provision of public services relative to the centralized solution. Less noted in the literature are the fiscal externalities associated with taxation of bases that are not co-occupied. If the two tax bases are substitutes then it is a positive externality; if complements, a negative fiscal externality. I begin, then, by illustrating the nature of the fiscal externalities associated with taxes in the context of this model. First I focus on the tax rates each government chooses, that is, the first-order conditions with respect to tax rates for the two levels of governments. Next I derive the first-order conditions for the tax bases when the two governments can choose both their tax bases. These first-order conditions are then used to determine the simultaneous equilibrium in tax rates and bases.

2.2.1 Local tax policy

Each locality maximizes its residents' welfare by choosing its tax rate given the tax rates and bases of the other localities and the state government. Its choice of tax rate also depends on the extent of its tax base. Formally, the local government's problem is to choose τ_l to maximize (2.5). Then in Nash equilibrium, the first-order condition for the local tax rate (τ_l) can be expressed as

$$W^l_{\tau_l} = -1 + \mathrm{MRS}_l \left[1 + \tau_l^* \left(\epsilon_{11} + (k_l + k_{ls})\epsilon_{21}\right)\right] = 0 \; or \; \mathrm{MRS}_l = \frac{1}{D_L} \qquad (2.9)$$

where τ_l^* is the equilibrium local tax rate, $\mathrm{MRS}_l \equiv \frac{\frac{\partial U^l}{\partial g_l}}{\frac{\partial V}{\partial x}}$ is the marginal rate of substitution between the local public good and private goods, and $D_L = \left[1 + \tau_l^* \left(\epsilon_{11} + (k_l + k_{ls})\epsilon_{21}\right)\right] > 0$. Then $\frac{1}{D_L}$ is the local government's marginal cost of funds (MCF_l), and the local government's rule is simply to set its tax rate to equate MRS_l with MCF_l. As $0 < D_L < 1$ then $\mathrm{MRS}_l > 1$.

The marginal external cost $\left(\mathrm{MEC}_{\tau_l}\right)$ associated with the tax of a single locality is

$$\mathrm{MEC}_{\tau_l} = \frac{1}{n}\mathrm{MRS}_s \frac{dg_s}{d\tau_l} = \frac{1}{n}\tau_s \mathrm{MRS}_s X_s \left[\frac{k_{ls}}{(k_{ls} + k_s)}\frac{x_{ls}}{\overline{x}_s}\epsilon_{11} + (k_l + k_{ls})\,\epsilon_{21}\right] \qquad (2.10)$$

where $\mathrm{MRS}_s \equiv \frac{\frac{\partial U^s}{\partial g_s}}{\frac{\partial V}{\partial x}}$ is the marginal rate of substitution between the state public good and private goods. Then from (2.10) the sign of MEC_{τ_l} , the welfare change associated with a change in level of the state public good is the sign of $\frac{\mathrm{d}g_s}{\mathrm{d}\tau_l}$ and, as (2.10) shows, may be positive or negative depending on the extent that the two governments share a tax base and whether commodities are complements or substitutes.

2.2.2 State tax policy

In the Nash equilibrium the first-order condition for the state tax rate (τ_s) can be expressed as

$$\mathrm{MRS}_s = \frac{1 - \alpha \mathrm{MRS}_l \frac{\mathrm{d}g_l}{\mathrm{d}\tau_s}}{D_S} \tag{2.11}$$

where τ_s^* is the equilibrium state tax rate and $1 > D_s = \left[1 + \tau_s^* \left(\epsilon_{11} + (k_s + k_{ls})\, \epsilon_{21}\right)\right] > 0$. Analogous to local governments, $\frac{1}{D_S}$ is the marginal cost of funds for the state government. As seen in (2.11) if $\alpha > 0$ the state considers the effects of its tax rate on local revenues. If $\alpha = 0$, analogous to the case with local governments, $\mathrm{MRS}_s > 1$.

While the state government may internalize some of the impact of its tax rate on local revenues, the extent, though not the sign, of the external cost associated with an increase in the state tax depends on the value of α with

$$\begin{aligned} \mathrm{MEC}_{\tau_s} &= (1-\alpha)\, \mathrm{MRS}_l \frac{\mathrm{d}g_l}{\mathrm{d}\tau_s} \\ &= (1-\alpha)\, \mathrm{MRS}_l \tau_l X_l \left[\frac{k_{ls}}{(k_{ls} + k_l)} \frac{x_{ls}}{\overline{x}_l} \epsilon_{11} + (k_s + k_{ls})\, \epsilon_{21} \right]. \end{aligned} \tag{2.12}$$

As with MEC_l, the sign of MEC_s depends on whether local revenue increases or decreases with an increase in the state tax rate.

Then summarizing the key results from this section: (1) the marginal external tax costs, MEC_{τ_l} and MEC_{τ_s} may be positive or negative depending on the extent of the overlap in tax bases and cross-price elasticities; (2) if the state government fully considers the impacts of its policies on local government policies there is no fiscal externality associated with the state tax $\left(\mathrm{MEC}_{\tau_s} = 0\right)$; and (3) the relative magnitude of the marginal rates of substitution for the two public goods is ambiguous. A more formal statement of these results is found as a proposition in Supplementary Appendix A.2.1.

2.3 Fiscal externalities and the choice of tax base

In the USA, the choice of tax base, that is what local governments can tax, is generally at the discretion of state, not local, governments. While this may be the case, it is still useful to examine what tax base local governments would choose if given the option. Here I begin by considering the problem facing local and state governments when they can choose both their tax rate and base.

2.3.1 Local tax base

Each locality maximizes its residents' welfare by choosing both its tax rate and tax base given the tax rates and bases of the other localities and the state government. As discussed, the optimal tax rate is given by (2.9). In the Nash equilibrium the local welfare-maximizing tax base $\left(\overline{K}_l^*\right)$ satisfies the first-order condition,

$$\left(1-\overline{K}_l^*\right) W_{\overline{k}_l}^l=\left(1-\overline{K}_l^*\right) \tau_l^*\left[\left(\mathrm{MRS}_l-1\right) x_z+\tau_l^* \mathrm{MRS}_l X_l \epsilon_{21}\right]=0, \quad (2.13)$$

where $z = ls, (l)\ \ if\ \ \overline{K}_l > (<) \overline{K}_s$. As $\mathrm{MRS}_l - 1 > 0$ at the optimal tax rate, as is evident from (2.13) if $\epsilon_{21} > 0$ it must be the case that (2.13) is only satisfied when $\overline{K}_l^* = 1$. The local government chooses to tax the entire base. As shown in Supplementary Appendix A.2, if $\epsilon_{21} < 0$ it is still optimal to tax the entire base.

2.3.2 State tax base

The optimal tax base for the state is characterized by the first-order condition,

$$\overline{K}_s^* W_{k_s}^s=\overline{K}_s^* \tau_s^*\left[\begin{array}{c}\left(\mathrm{MRS}_s-1\right) x_z+\mathrm{MRS}_s \tau_s^* X_s \epsilon_{21} \\ +\alpha \mathrm{MRS}_l \tau_l^*\left(D \epsilon_{11}+X_l \epsilon_{21}\right)\end{array}\right]=0, \quad (2.14)$$

where $z = ls\ (s)$ and $D = 1(0)\ \ if\ \ \overline{K}_l > (<) \overline{K}_s$. Using the first-order condition for the state tax rate (2.11) we can express (2.14) as

$$\overline{K}_s^* W_{k_s}^s=\overline{K}_s^* \tau_s^* \overline{x}_s\left[\left(\mathrm{MRS}_s-1\right)\left(\frac{x_z}{\overline{x}_s}-1\right)-\mathrm{MRS}_s \tau_s^* \epsilon_{11}+\alpha \mathrm{MRS}_l \tau_l^* D\left(\frac{k_s}{k_s+k_{ls}}\right) \frac{x_{ls}}{\overline{x}_s} \epsilon_{11}\right]=0 \quad (2.15)$$

If the state ignores any impacts expansion of its tax base has on local revenues ($\alpha = 0$) then, as is the case with local governments, the state will choose to tax the entire base $\left(\overline{K}_s^* = 0\right)$.

Less obvious is the case when $\alpha = 1$ and the state fully considers the impact on local revenues when choosing both its tax rate and tax base. The state faces a trade-off when expanding its tax base—it lowers the marginal cost of funds associated with any state tax rate but also reduces local revenues. Intuitively, the gain in social welfare of an increase in a tax base, absent other distorting taxes, is equal to $-\mathrm{MRS}\tau\epsilon_{11}$. That the impact in (2.15) is $(\mathrm{MRS}_s - 1)\, D\left(\frac{x_z}{\overline{x}_s} - 1\right) - \mathrm{MRS}_s \tau_s^* \epsilon_{11}$ reflects the fact that when the two tax bases partially overlap, the addition to the tax base of another commodity (x_{ls}) is less than the average tax base per commodity in the existing base ($\overline{x}_s$). However, in the Nash equilibrium the local government will tax the entire base, making $k_s = 0$. In this case then from inspection of (2.15) $x_z = \overline{x}_s$ when $k_s = 0$ making $W_{k_s}^s\Big|_{k_s=0} = -\tau_s^{*2} V_y \mathrm{MRS}_s \epsilon_{11} > 0$ and it is clearly optimal for the state to tax the entire base. Implications of (2.13) and (2.14) are summarized in the proposition below.

Proposition 1 *Assume the local and state governments independently choose their tax bases.*

a) In equilibrium both levels of government tax the entire base.
b) When the state government maximizes social welfare ($\alpha = 1$) *in equilibrium* $MRS_s > MRS_l$*; when* $\alpha < 1$ *the relationship between* MRS_s *and* MRS_l *is not obvious.*

That the $\mathrm{MRS}_s > \mathrm{MRS}_l$ when the state government maximizes social welfare ($\alpha = 1$) is a result of the fact that the state government considers the impact its taxing decision has on local revenues, while local governments do not consider the effects of their taxes on state revenues. The relationship between MRS_s and MRS_l can be obtained by subtracting the first-order condition for the local tax rate, (2.9), from that of the state (2.11) to obtain

$$\mathrm{MRS}_s = \left[\frac{1 + (1-\alpha)\,\tau_l^*\,(\epsilon_{11} + \epsilon_{21})}{1 + \tau_s^*\,(\epsilon_{11} + \epsilon_{21})}\right]\mathrm{MRS}_l. \tag{2.16}$$

2.3.3 A numerical example

Table 2 provides a numerical example of the policies chosen by a single, central government and those chosen when the state and local governments act independently and both tax the entire base ($k_{ls} = 1$). The results in the table are based on demand for the commodities given by the log-linear demand function,

$$\begin{aligned} ln(x(k)) = \epsilon_{11} ln(q(k)) + k_l \epsilon_{21} ln(1+\tau_l) + k_s \epsilon_{21} ln(1+\tau_s) \\ + k_{ls}\epsilon_{21} ln(1+\tau_l+\tau_s) \end{aligned} \tag{2.17}$$

where $q(k)$ is the gross (after-tax) price of commodity k and ϵ_{11} and ϵ_{21} are, respectively, the own-price and cross-price elasticities of demand.[9] The utility from the public services is given by

$$\beta_j g_j^{\gamma_j}, \quad j = l, s. \tag{2.18}$$

Income is normalized to unity and 85% of commodities are subject to taxation ($k_l + k_{ls} + k_s = 1,\ K = 1.2$). Three alternative combinations of elasticity are used: ($\epsilon_{11} = -1, \epsilon_{21} = 0$), ($\epsilon_{11} = -0.5, \epsilon_{21} = -0.42$), and ($\epsilon_{11} = -1.5, \epsilon_{21} = 0.42$).[10] As well, the public good utility functions are parameterized to give three efficient levels of public goods where $\mathrm{MRS}_l = \mathrm{MRS}_s = 1$: a) $g_l^* = g_s^* = .1$, b) $g_l^* = .15,\ g_s^* = .05$, and $g_l^* = .05,\ g_s^* = .15$ which can be interpreted as income shares for the public goods.

[9] I am being loose with notation—in (2.17) ϵ_{11} and ϵ_{21} are elasticities, while elsewhere they refer to "percentage change." In the numerical examples, the elasticity is constant and the percentage change varies with $\frac{\mathrm{d}x_j}{\mathrm{d}q(j)} = \frac{\epsilon_{11}}{q(j)}$ with $q(j)$ being the gross (after-tax) price for the tax base and with $\frac{\mathrm{d}x_i}{\mathrm{d}q(j)}$, $j \neq i$ defined analogously.

[10] The cross-price elasticities satisfy the Cournot adding-up restriction of $\epsilon_{21} = -\frac{1}{K}(1+\epsilon_{11})$ with $K = 1.2$.

Table 2 Tax and public service policies for centralized policy and independent policy with shared tax base ($k_{ls} = 1$)

	$\epsilon_{11} = -1,\ \epsilon_{21} = 0$						$\epsilon_{11} = -0.5,\ \epsilon_{21} = -0.42$						$\epsilon_{11} = -1.5,\ \epsilon_{21} = 0.42$					
Local public good, efficient	0.1		0.15		0.05		0.1		0.15		0.05		0.1		0.15		0.05	
State public good, efficient	0.1		0.05		0.15		0.1		0.05		0.15		0.1		0.05		0.15	
Centralized policy																		
Tax rate	0.171		0.171		0.171		0.173		0.173		0.173		0.169		0.169		0.169	
Local public good	0.073		0.109		0.036		0.075		0.112		0.037		0.071		0.107		0.036	
State public good	0.073		0.036		0.109		0.075		0.037		0.112		0.071		0.036		0.107	
$MRS_l = MRS_s$	1.171		1.171		1.171		1.156		1.156		1.156		1.185		1.185		1.185	
Independent tax policy																		
α	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
Local tax rate	0.101	0.099	0.140	0.138	0.054	0.054	0.100	0.099	0.140	0.139	0.054	0.053	0.102	0.099	0.139	0.138	0.055	0.054
State tax rate	0.101	0.086	0.054	0.043	0.140	0.128	0.100	0.086	0.054	0.043	0.140	0.129	0.102	0.086	0.055	0.043	0.139	0.127
State tax + Local tax	0.202	0.185	0.194	0.181	0.194	0.182	0.200	0.185	0.194	0.181	0.194	0.183	0.203	0.185	0.195	0.181	0.195	0.182
Local public good	0.084	0.084	0.117	0.117	0.046	0.046	0.085	0.085	0.119	0.119	0.046	0.046	0.083	0.085	0.115	0.115	0.045	0.045
State public good	0.084	0.072	0.046	0.036	0.117	0.109	0.085	0.073	0.046	0.037	0.119	0.111	0.083	0.073	0.045	0.036	0.115	0.106
MRS_l	1.092	1.092	1.132	1.132	1.048	1.048	1.087	1.086	1.123	1.123	1.046	1.046	1.097	1.086	1.142	1.142	1.050	1.050
MRS_s	1.092	1.177	1.048	1.175	1.132	1.175	1.087	1.167	1.046	1.169	1.123	1.163	1.097	1.167	1.050	1.181	1.142	1.188
$\frac{MRS_s}{MRS_l}$	1.000	1.078	0.925	1.038	1.081	1.122	1.000	1.075	0.931	1.041	1.074	1.112	1.000	1.075	0.919	1.034	1.088	1.131

Table 2 first reports the results when there is a centralized policy with a single tax and revenue is allocated between the two public goods to ensure that $\mathrm{MRS}_l = \mathrm{MRS}_s$. As the marginal cost of funds exceeds unity, in all cases the level of the public good is less than the efficient level.

The outcomes when the two levels of government pursue independent tax policies satisfying (2.9) and (2.11) with a shared tax base are reported in the table below those for the centralized government. Results are for both the cases in which the state government ignores the impacts of its policies on local revenues ($\alpha = 0$) and when it does not ($\alpha = 1$). When $\alpha = 0$ both governments "over-provide" the public services relative to the centralized government outcome. While the local public good level and local tax rate are relatively unchanged with $\alpha = 1$, the state public good and tax rate both decrease with the public good level approximating the level with the centralized government policy.

3 Optimal tax base division and co-occupancy

As shown in the preceding section, both levels of government will tax the entire tax base if given the option and changes in their tax bases as well as their tax rates generate fiscal externalities. Here I address the question of what is the social-welfare-maximizing division of the tax base between the two levels of government. To do this, I first address the question of how the tax base should be divided in the absence of co-occupancy. Then using this division of the tax base in the absence of co-occupancy as a starting point I then examine the optimality of co-occupancy.

If the state government fully considers the impacts of its tax policy on local revenues ($\alpha = 1$) and if it were also to have the authority to determine the extent of both tax bases, its choices would be welfare-maximizing. However, if $\alpha < 1$, the state government does not fully consider the impacts of its tax policy on local revenues and if the state government were to determine the tax bases for both levels of government it would not choose the welfare-maximizing division. As I wish to investigate the welfare-maximizing division of the tax base, in this case I assume a third-party (federal government or "planner") chooses the tax bases to maximize social welfare.

The timing of the division of the tax base, relative to the setting of the state and local tax rates, also needs to be determined. My focus will be on a Nash equilibrium in which the state and local governments choose their tax rates at the same time as the planner chooses the division of the tax base. In Supplementary Appendix A.3.4 I discuss a two-stage game, along the lines of Hoyt and Jensen (1996) and Koethenbuerger (2008), in which the federal government chooses the division of the tax base in the first stage of the game and the state and local governments simultaneously set their tax rates in the second stage and find that the results are qualitatively similar to the simultaneous game.

While initially I restrict the state government to apply the same tax rate throughout its tax base, it may have an incentive to differentially tax the segment of its tax base that it shares with local government differently than the component that it alone taxes. This is discussed in Sect. 3.3.

3.1 The optimal division of the tax base

The optimal division of the tax base in the absence of co-occupancy solves

$$\underset{\overline{K}_l}{\text{Max}}\ W\left[\tau_l, \tau_s, \overline{K}_l\right] = \int_0^1 V(k)\,\mathrm{d}k + U^s\left(g_s\left(\tau_l, \tau_s, \overline{K}_l\right)\right) + \alpha U^l\left(g_l\left(\tau_l, \tau_s, \overline{K}_l\right)\right) \tag{3.1}$$

where, given no co-occupancy, $k_l = \overline{K}_l$ and $k_s = 1 - \overline{K}_l$. Then the optimal division of the tax base, $\overline{K}_l^*$, that satisfies the first-order condition for (3.1) can be expressed as

$$\begin{aligned}&\tau_l^*\left[\underset{(a)}{\left(-1 + \text{MRS}_l\left(1 + \tau_l^* k_l \epsilon_{21}\right)\right) x_l} + \underset{(b)}{\text{MRS}_s \tau_s^* k_s x_s \epsilon_{21}}\right] \\ &= \tau_s^*\left[\underset{(c)}{\left(-1 + \text{MRS}_s\left(1 + \tau_s^* k_s \epsilon_{21}\right)\right) x_s} + \underset{(d)}{\text{MRS}_l \tau_l^* k_l x_l \epsilon_{21}}\right]\end{aligned} \tag{3.2}$$

where τ_l^* and τ_s^* are the rates that satisfy (2.9) and (2.11). Derivation of (3.2) and other equations as well as proofs of propositions in this section are found in Supplementary Appendix A.3. The expansion of the local tax base and contraction of the state tax base directly increases local revenue by $\tau_l x_l$ and indirectly by affecting the price of $x\left(\overline{K}_l\right)$, now taxed at a rate of τ_l rather than τ_s. As well, the tax has a direct impact through its impact on the price of $x\left(\overline{K}_l\right)$. These impacts are found in term (a) of (3.2). Term (b) is the impact of adding $x\left(\overline{K}_l\right)$ to the local tax base on state tax revenues. Then the optimal division of the tax base must be such that the impact on utility of the expansion of the local tax base is exactly offset by the impact of the equal reduction of the state tax base, terms (c) and (d) of (3.2). Rearranging terms in (3.2) yields

$$\underset{(a)}{\left(\text{MRS}_l - 1\right)\tau_l^* x_l - \left(\text{MRS}_s - 1\right)\tau_s^* x_s} + \underset{(b)}{\left(\tau_l^* - \tau_s^*\right)\left(\text{MRS}_l \tau_l^* k_l x_l + \text{MRS}_s \tau_s^* k_s x_s\right)\epsilon_{21}} = 0. \tag{3.3}$$

Examination of Eq. (3.3) provides an additional characterization of the optimal division of the tax base. From (3.3) we can see that if commodities are substitutes $(\epsilon_{21} > 0)$ the sign of term (b) is the sign of $\tau_l - \tau_s$. Then if, for example, $\tau_l > \tau_s$ it must be the case that term (a) is negative, requiring $\text{MRS}_l < \text{MRS}_s$. If $\tau_l(.5) > (<)\tau_s(.5)$ then $W_{\overline{k}_l}\Big|_{\overline{k}_l=.5} > (<)\,0$ and the optimal division of the tax base must be $\overline{K}_l^* > (<).5$. With $\epsilon_{21} > 0$ and $\overline{K}_l^* > (<).5$ (3.3) can only be satisfied when the optimal tax rates are $\tau_l^* > (<)\tau_s^*$.

When commodities are complements or $\alpha \neq 0$, the relationship between the relative tax rates and the relative MRS is indeterminate—with $\tau_l > \tau_s$ it is possible to have $\text{MRS}_l < \text{MRS}_s$ or $\text{MRS}_l > \text{MRS}_s$ and satisfy (3.3). This being the case, it is possible for (3.3) to be satisfied with no obvious relationship between the equilibrium tax rates $\left(\tau_l^*, \tau_s^*\right)$, the associate MRS, and the relative tax rates when the base is evenly split.

Proposition 2 *In the absence of co-occupancy, the optimal division* $\left(\overline{K}_l^*\right)$ *of the tax base can be characterized by the following:*

a) If the state ignores the impact of its tax rate policy on local revenues ($\alpha = 0$) *and i)* $\tau_l(.5) > (<)\tau_s(.5)$ *then* $\overline{K}_l^* > (<).5$*; ii)* $\epsilon_{21} > 0$ *and* $\tau_l(.5) > (<)\tau_s(.5)$ *then* $\tau_l\left(\overline{K}_l^*\right) > (<)\tau_s\left(\overline{K}_l^*\right)$ *and* $MRS_l\left(\overline{K}_l^*\right) < (>)MRS_s\left(\overline{K}_l^*\right)$*;*

b) At $\overline{K}_l^*$*,* $MRS_l\tau_l\left(\overline{K}_l^*\right) \neq MRS_s\tau_s\left(\overline{K}_s^*\right)$ *if* $\epsilon_{21} \neq 0$.

c) At $\overline{K}_l^*$*: i) if* $\epsilon_{21} = 0$*, a (marginal) change in either the local or state tax rate has no impact on social welfare; ii) if* $\epsilon_{21} > (<)\,0$ *and* $\alpha = 0$*, an increase (decrease) in the local or state tax rate will increase social welfare; iii) if* $\epsilon_{21} > (<)\,0$*, and* $\alpha = 1$ *an increase (decrease) in the local tax rate will increase social welfare, while an increase in the state tax rate will have no impact on social welfare.*

Proof of the proposition is found in Supplementary Appendix A.3.2. Again, abstracting from the technical aspects of the proposition, there are several important implications for the consideration of the optimality of co-occupancy. As stated in *Part a)* when $\alpha = 0$ the relative tax rates and marginal rates of substitutions are inversely related. That $\mathrm{MRS}_l\tau_l\left(\overline{K}_l^*\right) \neq \mathrm{MRS}_s\tau_s\left(\overline{K}_l^*\right)$ if $\epsilon_{21} \neq 0$ (*Part b*) means that the additional welfare associated with an increase in the public goods obtained from the expansion of the state and local tax bases are not equal at the optimal division of the tax base. Finally, *Part c*) provides relationships between the division of the tax base and the fiscal externality from increases in the tax rates. As suggested by the proposition, the division of the tax base will not eliminate the fiscal externalities associated with the two tax rates when commodities have nonzero cross-price elasticities. In the case of gross substitutes ($\epsilon_{21} > 0$), it may change the fiscal externality from being negative with co-occupancy $\left(\mathrm{MEC}_{\tau_j} = \mathrm{MRS}_i\tau_i X_i\left[\epsilon_{11} + \left(k_j + k_{ls}\right)\epsilon_{21}\right] < 0\right)$ to being positive with no co-occupancy $\left(\mathrm{MEC}_{\tau_j} = \mathrm{MRS}_i\tau_i X_i\left(k_j + k_{ls}\right)\epsilon_{21}\right)$. This, in turn, means that taxes also change from being "too" high as a result of a negative fiscal externality to being "too" low that is, below the welfare-maximizing rates as the fiscal externality is now positive.

3.1.1 A numerical example

Table 3 presents the results of simulations solving (3.3) using the same specifications for commodity and public service demand used in Sect. 2.3.3. With $\epsilon_{11} = -1$ and $\epsilon_{21} = 0$ (*Panel A*), the tax base is allocated to ensure that the state and local tax rates and marginal rates of substitution for the public goods are equal and the tax and public service policies are the same as those of the centralized government. As $\epsilon_{21} = 0$ there is no external cost in the absence of co-occupancy and, as a result, no difference in the policy when $\alpha = 0$ and when $\alpha = 1$.

In *Panel B* the results with $\epsilon_{11} = -0.5$ and $\epsilon_{21} = -0.42$ are reported. In this case, there are negative fiscal externalities from the taxes and both public services are over-provided relative to the levels found with centralized government. The tax base is greater and the tax rate lower for the level of government that has the greater demand for the public good when $\alpha = 0$. However, when $\alpha = 1$ the state tax rate is lower than

Table 3 Tax and public service policies for optimal division of the tax base ($k_{ls} = 0$)

	A $\epsilon_{11} = -1,\ \epsilon_{21} = 0$						B $\epsilon_{11} = -0.5,\ \epsilon_{21} = -0.42$						C $\epsilon_{11} = -1.5,\ \epsilon_{21} = 0.42$					
Local public good, efficient	0.1		0.15		0.05		0.1		0.15		0.05		0.1		0.15		0.05	
State public good, efficient	0.1		0.05		0.15		0.1		0.05		0.15		0.1		0.05		0.15	
α	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
Local tax rate	0.171	0.171	0.171	0.171	0.171	0.171	0.185	0.215	0.167	0.185	0.244	0.262	0.159	0.157	0.165	0.163	0.151	0.150
State tax rate	0.171	0.171	0.171	0.171	0.171	0.171	0.185	0.152	0.244	0.156	0.167	0.159	0.159	0.171	0.151	0.170	0.165	0.170
Local tax base	0.500	0.500	0.750	0.750	0.250	0.250	0.500	0.424	0.807	0.722	0.193	0.178	0.500	0.508	0.744	0.753	0.256	0.259
State tax base	0.500	0.500	0.250	0.250	0.750	0.750	0.500	0.576	0.193	0.278	0.807	0.822	0.500	0.492	0.256	0.247	0.744	0.741
Local public good	0.073	0.073	0.109	0.109	0.036	0.036	0.079	0.077	0.116	0.115	0.039	0.039	0.068	0.068	0.104	0.104	0.033	0.034
State public good	0.073	0.073	0.036	0.036	0.109	0.109	0.079	0.076	0.039	0.038	0.116	0.113	0.068	0.071	0.033	0.035	0.104	0.106
MRS_l	1.171	1.171	1.171	1.171	1.171	1.171	1.124	1.136	1.136	1.143	1.128	1.135	1.215	1.211	1.202	1.199	1.224	1.222
MRS_s	1.171	1.171	1.171	1.171	1.171	1.171	1.124	1.147	1.128	1.150	1.136	1.150	1.215	1.190	1.224	1.189	1.202	1.189
$\frac{\mathrm{MRS}_s}{\mathrm{MRS}_l}$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.010	0.993	1.006	1.007	1.013	1.000	0.982	1.018	0.991	0.982	0.973
MEC_l	0.000	0.000	0.000	0.000	0.000	0.000	−0.032	−0.031	−0.016	−0.016	−0.045	−0.044	0.031	0.031	0.015	0.016	0.047	0.047
MEC_s	0.000	0.000	0.000	0.000	0.000	0.000	−0.037	−0.036	−0.055	−0.054	−0.018	−0.017	0.035	0.035	0.053	0.053	0.017	0.017
$\mathrm{MRS}_s\tau_s - \mathrm{MRS}_l\tau_l$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	−0.075	0.089	−0.034	−0.089	−0.121	0.000	0.020	−0.020	0.010	0.020	0.030
$W_{\overline{K}_l}\Big\vert_{k_{ls}=0}$	0.000	0.000	0.000	0.000	0.000	0.000	−0.037	−0.011	−0.047	−0.006	−0.025	−0.015	0.035	0.022	0.031	0.011	0.039	0.033
$W_{\overline{K}_s}\Big\vert_{k_{ls}=0}$	0.000	0.000	0.000	0.000	0.000	0.000	−0.037	−0.061	−0.025	−0.065	−0.047	−0.056	0.035	0.049	0.039	0.060	0.031	0.038

the local tax rate and the state share of the tax base increases relative to when $\alpha = 0$. For example, when $g_l^* = g_s^* = .1$ and $\alpha = 0$ both levels of government set a tax rate of 0.185 and the tax base is evenly split ($k_l = k_s = .5$) but when $\alpha = 1$, $k_l = .424$ and $k_s = .576$ while $\tau_l = 0.215$ and $\tau_s = 0.152$. Finally, in *Panel C* with $\epsilon_{11} = -1.5$ and $\epsilon_{21} = 0.42$, the division of the tax base does not vary much from that in Panel A. One noticeable difference in the outcomes in *Panel B* and *Panel C* is that with $\alpha = 1$ in *Panel B*, the state tax exceeds the local tax, while this is reversed in *Panel C* with the state tax base smaller in *Panel B* than for comparable public service demands than in *Panel C*.

3.2 Optimal co-occupancy of tax bases

That the tax rates and marginal rates of substitutions for the public services are not equal for the two levels of government and tax rate increases or decreases can enhance social welfare suggests at least the possibility that co-occupancy may be desirable. Below I consider whether and under what conditions co-occupancy may be socially optimal.

The problem facing the planner is

$$\underset{\overline{K}_l, \overline{K}_s}{\text{Maximize}}\ W\left(\overline{K}_l, \overline{K}_s, \tau_s, \tau_l\right) = \int_0^1 V(q(k)\mathrm{d}k + U^s\left(g_s\left(\overline{K}_l, \overline{K}_s, \tau_s, \tau_l\right)\right) + U^l\left(g_l\left(\overline{K}_l, \overline{K}_s, \tau_s, \tau_l\right)\right) \tag{3.4}$$

Then the first-order conditions with respect to the tax bases when $k_{ls} > 0$ can be expressed as

$$\left(1 - \overline{K}_l\right) W_{\overline{K}_l} = \left(1 - \overline{K}_l\right) V_y \tau_l^* x_{ls} \begin{bmatrix} -1 + \text{MRS}_l\left(1 + \tau_l^*(k_{ls} + k_l)\frac{\overline{x}_l}{x_{ls}}\epsilon_{21}\right) \\ + \text{MRS}_s \tau_s^*\left(\epsilon_{11} + (k_s + k_{ls})\frac{\overline{x}_s}{x_{ls}}\epsilon_{21}\right) \end{bmatrix} = 0 \text{ and} \tag{3.5a}$$

$$\overline{K}_s W_{\overline{K}_s} = \overline{K}_s V_y \tau_s^* x_{ls} \begin{bmatrix} -1 + \text{MRS}_s\left(1 + \tau_s^*(k_{ls} + k_s)\frac{\overline{x}_s}{x_{ls}}\epsilon_{21}\right) \\ + \text{MRS}_l \tau_l^*\left(\epsilon_{11} + (k_l + k_{ls})\frac{\overline{x}_l}{x_{ls}}\epsilon_{21}\right) \end{bmatrix} = 0. \tag{3.5b}$$

As discussed earlier, to determine whether it is optimal to have co-occupied tax bases I first evaluate the gains from co-occupancy when the tax base is optimally divided in the absence of co-occupancy as characterized by (3.3). Now to consider the impact of an increase in the local tax base on social welfare when there is no co-occupancy $\left(\overline{K}_l = \overline{K}_s\right)$, subtract $W_{\tau_l}^l\Big|_{\tau_l^*} = 0$ (2.9) from (3.5a) and evaluate with $k_{ls} = 0$ to obtain

$$W_{\overline{K}_l} = \tau_l^* x_{ls}\left[\left(\text{MRS}_l \tau_l^* - \text{MRS}_s \tau_s^*\right)(-\epsilon_{11}) + \left[\text{MRS}_l \tau_l^* k_l\left(\frac{x_l}{x_{ls}} - 1\right) + \text{MRS}_s \tau_s^* k_s \frac{x_s}{x_{ls}}\right]\epsilon_{21}\right]. \tag{3.6a}$$

Analogously, subtracting the first-order condition for the state tax rate (2.11) from (2.14) and evaluating at $k_{ls} = 0$ gives

$$- W_{\overline{K}_s}\Big|_{k_{ls}=0} = \tau_s^* x_{ls} \left[\begin{array}{c} \left(\mathrm{MRS}_s \tau_s^* - \mathrm{MRS}_l \tau_l^*\right)(-\epsilon_{11}) \\ + \left[\mathrm{MRS}_s \tau_s^* k_s \left(\frac{x_s}{x_{ls}} - 1\right) + \mathrm{MRS}_l \tau_l^* \left(k_l \frac{x_l}{x_{ls}} - \alpha k_s \frac{x_l}{x_{ls}}\right)\right] \epsilon_{21} \end{array} \right]. \tag{3.6b}$$

From (3.6a)–(3.6b), results about the optimal co-occupancy of the tax base can be obtained. These results are summarized in the proposition below.

Proposition 3 *Let τ_l^*, τ_s^*, and $\overline{K}_l^*$ represent the values of the tax rates and the division of the tax base in the absence of co-occupancy that satisfy (2.9), (2.11), and (3.3). Then:*

a) Necessary conditions for the optimal division of the tax base to be such that some segment of the tax base is shared by both levels of government (co-occupied) are that either (3.6a) or (3.6b) are positive.

b) If $\epsilon_{21} > 0$ either $W_{\overline{k}_l}\Big|_{k_{ls}=0} > 0$ or $-W_{\overline{k}_s}\Big|_{k_{ls}=0} > 0$ and it is optimal to co-occupy some share of the tax base.

c) If $\epsilon_{21} < 0$ then a sufficient condition for it not to be optimal to co-occupy the tax base is $\left(\tau_l^* x_{ls} - \tau_s^* x_s\right) + k_s \tau_l^* \tau_s^* x_s \left(\frac{x_{ls}}{x_s} - 1\right) \epsilon_{21} < 0$ and $\left(\tau_s^* x_{ls} - \tau_l^* x_l\right) + k_l \tau_l^* \tau_s^* x_l \left(\frac{x_{ls}}{x_l} - 1\right) \epsilon_{21} < 0$.

d) Let $\widetilde{K}_l$ and $\widetilde{K}_s$ denote the optimal division of the tax base. Then if it is optimal to have co-occupancy either: i) one or both governments will tax the entire base ($\widetilde{K}_l = 1$ and $\widetilde{K}_s = 0$ $(k_{ls} = 1)$, $\widetilde{K}_l = 1$ and $\widetilde{K}_s > 0$ $(k_{ls} < 1,\ k_l > 0,\ k_s = 0)$, or $\widetilde{K}_l < 1$ and $\widetilde{K}_s = 0$ $(k_{ls} < 1,\ k_l = 0,\ k_s > 0)$, or ii) each government taxes only part of the base $(0 < \widetilde{K}_s < \widetilde{K}_l < 1)$. In this case, at the optimal division of the tax base,

$$MRS_l + MRS_s \tau_s^* \epsilon_{11} = MRS_s + MRS_l \tau_l^* \epsilon_{11}. \tag{3.7}$$

Proof of part c) *is found in Supplementary Appendix A.3.3 with remainder of the* Proposition *discussed below. That it is optimal to have co-occupancy when the commodities are substitutes* ($\epsilon_{21} > 0$) (Part b) *follows immediately from the conditions determining whether any co-occupancy is optimal, (3.6a) and (3.6b). In these expressions, either the "direct" impact of an expansion of the local tax base, $MRS_l \tau_l^* - MRS_s \tau_s^*$, or the direct impact of the expansion of the state tax base, $MRS_s \tau_s^* - MRS_l \tau_l^*$, must be positive. Then if $\epsilon_{21} > 0$ the second term of both expressions, the impact on the segment of the other level's tax base that is not co-occupied, is positive, meaning that one of the two conditions, or possibly both must be positive.*

In *Part d ii)*, the condition to be satisfied when each government only taxes part of the base, (3.7), is a necessary condition for both of the first-order conditions with respect to the tax base, (3.5a) and (3.5b), to be satisfied for an interior solution. Intuitively, this is simply the condition that the marginal social cost of increasing the tax base, the benefits from the increase in its public service and the decrease in benefits from the other government's public service, must be equal for both levels of government. Note that if $\text{MRS}_s = \text{MRS}_l$ and $\tau_l = \tau_s$ this condition is satisfied. More generally, (3.7) suggests that the "indirect" effects of a marginal expansion of the two governments' base, the change in tax revenue from the existing bases for both governments, cancel each other out and the direct effects of expansion, the increase in the benefits from the public services for the expanding base (MRS_l, MRS_s) and the decrease in benefits for the other base ($\text{MRS}_l\tau_l$, $\text{MRS}_s\tau_s$) are the only relevant factors.

To provide some further intuition, consider evaluating (3.5a) at the optimal division of the tax base in the case of the state and local tax rates being equal and, by (3.3), $\text{MRS}_l = \text{MRS}_s$. Then $\text{MRS}_s\tau_s - \text{MRS}_l\tau_l = 0$ and (3.6a) and (3.6b) become

$$W_{\bar{k}_l}\Big|_{k_l^*} = \left[\text{MRS}_l\tau_l^* k_l\left(\frac{x_l}{x_{ls}} - 1\right) + \text{MRS}_s\tau_s^* k_s\frac{x_s}{x_{ls}}\right]\epsilon_{21} \quad \text{and} \tag{3.8a}$$

$$-W_{\bar{k}_s}\Big|_{k_l^*} = \left[\text{MRS}_s\tau_s^* k_s\left(\frac{x_s}{x_{ls}} - 1\right) + \text{MRS}_l\tau_l^*\left(k_l\frac{x_l}{x_{ls}} - \alpha k_s\frac{x_l}{x_{ls}}\right)\right]\epsilon_{21}. \tag{3.8b}$$

As discussed earlier, the state and local tax rates will not be equal at the optimal division of the base if $\epsilon_{21} \neq 0$ and this difference in the two tax rates accounts for the first terms in (3.6a) and (3.6b) in which there are additional welfare gains to expanding the base for which the "direct" impact ($\text{MRS}\tau$) is greatest. Intuitively, (3.8a) is the welfare impact of increases (decreases) in tax revenue in the segments of both the state and local tax base that are not co-occupied if $\epsilon_{21} > (<)\, 0$. This increase (decrease) in tax revenue is relative to that obtained by expanding the local tax base while reducing the state tax base by an equal amount. An analogous argument can be made for expansion of the state tax base and (3.8b).

3.2.1 A numerical example

In Table 3 the rows entitled $W_{\overline{K}_l}\Big|_{k_{ls}=0}$ and $W_{\overline{K}_s}\Big|_{k_{ls}=0}$ provide the evaluation of (3.6a) and (3.6b) for each case of the numerical example. Consistent with Proposition 3, when the tax base is optimally divided, co-occupancy is only welfare-increasing in the case in which commodities are substitutes ($\epsilon_{11} = -1.5,\ \epsilon_{21} = 0.42$). When the cross-price elasticity is zero (*Panel A*) $W_{\overline{K}_l}\Big|_{k_{ls}=0} = W_{\overline{K}_s}\Big|_{k_{ls}=0} = 0$ with both of these expressions being negative when the commodities are complements (*Panel B*).

Then in Table 4 the optimal tax base and tax rates are reported for the cases with $\epsilon_{11} = -1.5,\ \epsilon_{21} = 0.42$. With equal demands for the public services $\left(g_l^* = g_s^* = .1\right)$ it is optimal for both the state and local government to occupy the entire tax base, thereby replicating the policies of the independent governments. Given this symmetry, specifically with $\text{MRS}_l\tau_l^* = \text{MRS}_s\tau_s^*$ when $\alpha = 0$, this is consistent with equations (3.6a) and (3.6b) both being positive for all values of $\overline{K}_l$ and $\overline{K}_s$.

With asymmetric demands for the public service and $\alpha = 0$, the level of government with greater demand will have exclusive rights to tax some share of the base, while the level of government with a lower demand shares its entire base—thus with $g_l^* = .15$ and $g_s^* = .05$, $k_l = .453$, $k_{ls} = .547$ and $k_s = 0$, while the allocation of the tax

Table 4 Tax and public service policies with optimal co-occupancy

	$\epsilon_{11} = -1.5,\ \epsilon_{21} = 0.42$					
Local public good, efficient	0.1		0.15		0.05	
State public good, efficient	0.1		0.05		0.15	
α	0	1	0	1	0	1
Local tax rate	0.113	0.110	0.148	0.153	0.103	0.099
State tax rate	0.113	0.093	0.103	0.046	0.148	0.136
Local tax base (k_l)	0	0	0.453	0	0.000	0.000
Shared tax base (k_{ls})	1	1	0.547	1	0.547	0.562
State tax base (k_s)	0	0	0	0	0.453	0.438
Local public good	0.083	0.083	0.117	0.117	0.041	0.041
State public good	0.083	0.070	0.041	0.035	0.117	0.109
MRS_l	1.095	1.095	1.132	1.134	1.102	1.099
MRS_s	1.095	1.192	1.102	1.193	1.132	1.175
$\frac{\mathrm{MRS}_s}{\mathrm{MRS}_l}$	1.000	1.089	0.973	1.052	1.027	1.069
$\mathrm{MRS}_s\tau_s - \mathrm{MRS}_l\tau_l$	0.000	0.010	0.055	0.119	−0.055	−0.051

base is reversed when $g_l^* = .05$ and $g_s^* = .15$, $k_l = 0$, $k_{ls} = .547$ and $k_s = .453$. However, when $\alpha = 1$ the state government will tax the entire base. In this case, with $g_l^* = .15$ and $g_s^* = .05$ the local government will also tax the entire base but when $g_l^* = .05$ and $g_s^* = .15$, it is restricted to only approximately half of the base ($k_{ls} = .562$). Provision of both public services increases relative to that obtained when no co-occupancy is allowed (Table 3) with the state public service approximating the level obtained with centralized government and the local public service over-provided relative to the centralized outcome.

3.3 Optimal co-occupancy with differential state taxation

Thus far, the analysis has been restricted to considering uniform tax rates across both tax bases. I now consider the possibility that the state government can set different rates on the shared tax base (k_{ls}) and the base that it alone taxes (k_s). For the state government to have an incentive to set (substantially) different tax rates on the two segments of its base, it must be the case that $\alpha \neq 0$—it considers the impact of its tax policies on local revenues. For simplicity, let $\alpha = 1$ and the choice of the tax bases be that of the state government. Then the objective of the state government is

$$\underset{\tau_s^{ls},\, \tau_s^s,\, \overline{K}_l,\, \overline{K}_s}{\text{Maximize}}\ W\left(\overline{K}_l, \overline{K}_s, \tau_s^{ls}, \tau_s^s, \tau_l\right) = \begin{array}{l} \int_0^1 V(q(k)\mathrm{d}k + U^s\left(g_s\left(\overline{K}_l, \overline{K}_s, \tau_s^{ls}, \tau_s^s, \tau_l\right)\right) \\ \quad + U^l\left(g_l\left(\overline{K}_l, \overline{K}_s, \tau_s^{ls}, \tau_s^s, \tau_l\right)\right) \end{array} \tag{3.9}$$

where τ_s^{ls} is the state rate on the co-occupied segment and τ_s^s is the state rate on the segment it alone taxes. The first-order conditions for both tax rates are given by

$$W_{\tau_s^s} = \begin{array}{c} -1 + \text{MRS}_s\left[1 + \tau_s^s\left(\epsilon_{11} + k_s\epsilon_{21}\right)\right] \\ + \text{MRS}\tau_s^{ls}k_{ls}\frac{x_{ls}}{x_s}\epsilon_{21} + \text{MRS}_l\tau_l\left(k_l + k_{ls}\right)\frac{\overline{x}_l}{x_s}\epsilon_{21} \end{array} = 0, \text{ and} \tag{3.10a}$$

$$W_{\tau_s^{ls}} = \begin{array}{c} -1 + \text{MRS}_s\left[1 + \tau_s^{ls}\left(\epsilon_{11} + k_{ls}\epsilon_{21}\right)\right] \\ + \text{MRS}_s\tau_s^s k_s\frac{x_s}{x_{ls}}\epsilon_{21} + \text{MRS}_l\tau_l\left(\epsilon_{11} + \left(k_l + k_{ls}\right)\frac{\overline{x}_l}{x_{ls}}\epsilon_{21}\right) \end{array} = 0. \tag{3.10b}$$

Considering the optimal tax base, I assume that there is overlap in the two bases ($k_{ls} > 0$). Then an increase in $\overline{K}_l$ means a reduction in the base only taxed by the state (k_s) and a decrease in $\overline{K}_s$ means a reduction in the base only taxed by the localities (k_l). Then the optimal tax bases are determined by

$$\left(1 - \overline{K}_l\right)W_{\overline{K}_l} = \left(1 - \overline{K}_l\right)\left[\begin{array}{c} \left[\left(\text{MRS}_l - 1\right)\tau_l + \left(\text{MRS}_s - 1\right)\tau_s^{ls}\right]x_{ls} - \left(\text{MRS}_s - 1\right)\tau_s^s x_s + \\ \left[\text{MRS}_l\tau_l\left(k_l + k_{ls}\right)\overline{x}_l + \text{MRS}_s\left(\tau_s^{ls}k_{ls}x_{ls} + \tau_s^s k_s x_s\right)\right]\epsilon_{21}\left(\tau_l + \tau_s^{ls} - \tau_s^s\right) \end{array}\right] = 0 \tag{3.11a}$$

and

$$-\overline{K}_s W_{\overline{K}_s} = \left(1 - \overline{K}_s\right)\left[\begin{array}{c} \left[\left(\text{MRS}_l - 1\right)\tau_l + \left(\text{MRS}_s - 1\right)\tau_s^{ls}\right]x_{ls} - \left(\text{MRS}_l - 1\right)\tau_l x_l + \\ \left[\text{MRS}_l\tau_l\left(k_l + k_{ls}\right)\overline{x}_l + \text{MRS}_s\left(\tau_s^{ls}k_{ls}x_{ls} + \tau_s^s k_s x_s\right)\right]\epsilon_{21}\tau_s^{ls} \end{array}\right] = 0. \tag{3.11b}$$

Consider a possible solution with $\text{MRS}_s = \text{MRS}_l \equiv \text{MRS}$ and $\tau_l + \tau_s^{ls} = \tau_s^s$. In this case, both (3.10a) and (3.10b) are satisfied only if $\overline{K}_s = 0$, $(k_l = 0)$. The state taxes the entire base. As well, (3.11a) will be satisfied with equality regardless of the division of the tax base. Then when $\text{MRS}_s = \text{MRS}_l \equiv \text{MRS}$ and $\tau_l + \tau_s^{ls} = \tau_s^s$, (3.11b) becomes

$$-\overline{K}_s W_{\overline{K}_s} = \left(1 - \overline{K}_s\right)\left[\left(\text{MRS} - 1\right)\left[\left(\tau_l + \tau_s^{ls}\right)x_{ls} - \tau_l x_l\right] + \text{MRS}\left(\tau_l + \tau_s^{ls}\right)\epsilon_{21}\tau_s^{ls}\right] \tag{3.12}$$

Then with $\left(\tau_l + \tau_s^{ls}\right)x_{ls} - \tau_l x_l > (<)\, 0$ when $\tau_s^{ls} > (<)\ 0$, for the state government to tax the entire base $\left(-W_{\overline{K}_s} > 0\right)$ it must be the case that when commodities are substitutes (complements) ($\epsilon_{21} > (<)\ 0$) the state tax on the co-occupied segment of the tax base is positive (negative).

Thus, two solutions appear possible: 1) the local government does not tax the entire base $\left(\overline{k}_l < 1\right)$ and $\text{MRS}_l = \text{MRS}_s$, and $\tau_s^{ls} + \tau_l = \tau_s^s$; and 2) both the state and local governments tax the entire base $\left(\overline{K}_l = 1\right)$ with $\text{MRS}_l > \text{MRS}_s$. However, to satisfy the first-order conditions for both the state (3.10b) and the local tax rates (2.9) in the second solution it must be the case that $\text{MRS}_s > \text{MRS}_l$. Thus, a contradiction and only the first solution is feasible.

When $\text{MRS}_l = \text{MRS}_s$ and $\tau_s^{ls} + \tau_l = \tau_s^s$ we can subtract the first-order condition for the local tax rate (2.9) from (3.11b) to obtain

$$\tau_s^{ls}\left(\epsilon_{11} + k_{ls}\epsilon_{21}\right) + \tau_s^s k_s\epsilon_{21} = 0. \tag{3.13}$$

Equation (3.13) states that the state government chooses its tax rates and the tax base that the local government can tax (k_{ls}) so that the local tax rate generates no fiscal externality. These results are summarized in the following proposition:

Proposition 4 *If $\alpha = 1$ and the state can set different tax rates on the shared base (k_{ls}) and the base it alone taxes (k_s), its optimal tax rates and the division of the tax base are such that:*

a) The state will tax the entire base $\left(\overline{K}_s = 0\right)$ and localities will only tax some share of the base $\left(\overline{K}_l < 1\right)$;
b) $MRS_l = MRS_s$;
c) The combined state and local tax on the co-occupied share of the tax base equals the state tax rate on the share of the base it alone taxes $\tau_s^{ls} + \tau_l = \tau_s^s$;
d) The local tax rate generates no fiscal externality, that is, $\tau_s^{ls}\left(\epsilon_{11} + k_{ls}\epsilon_{21}\right) + \tau_s^s k_s \epsilon_{21} = 0$; and
e) The state tax rate on the co-occupied segment is greater (less) than zero $\left(\tau_s^{ls} > (<)\ 0\right)$ if cross-price elasticities are greater (less) than zero $(\epsilon_{21} > (<)\ 0)$.

Proposition 4 a) states the key distinction in results for co-occupancy when the state is restricted to a single tax rate across its entire tax base and when it can set differentiated tax rates on the co-occupied base (k_{ls}) and the base it alone taxes (k_s). Specifically, when the state government is restricted to a single tax rate the optimality of co-occupancy is only certain when commodities are gross substitutes; however, when it can tax the co-occupied base (k_{ls}) and the tax base it alone taxes (k_s) at different rates and it can set a negative tax rate (subsidy) on the co-occupied base, co-occupancy is always optimal. While Hoyt (2001) considers the overlapping tax bases and the possibility of negative tax rates (subsidies), it was in the context of exogenously determined tax bases. In that case, whether the state government applies a positive (negative) tax rate on a shared tax base was determined by whether $\text{MRS}_l > (<) \text{MRS}_s$. In this case, the state government is choosing the extent of the overlap in tax base and the state applies a positive (negative) tax rate on the shared base when commodities are substitutes (complements).

3.3.1 A numerical example

Using the same parameterization as found in the earlier simulations, Table 5 reports the solution to the equilibrium conditions described in Proposition 4. As co-occupancy and differential taxation are not necessary to obtain an equilibrium with $\text{MRS}_s = \text{MRS}_l$ and $\tau_l = \tau_s$ when $\epsilon_{21} = 0$, this case is not considered. Then in *Panel A,* the results with $\epsilon_{11} = -0.5$ and $\epsilon_{21} = -0.42$ are reported. As in part e) of Proposition 4, the state tax on the shared tax base is negative and quite significant. In contrast, in *Panel B* with results for $\epsilon_{11} = 0.5$ and $\epsilon_{21} = 0.42$, the tax on the shared tax is positive albeit small. The extent of the shared tax base varies with the demand for the public good and the cross-price elasticities with less of the base shared when commodities are complements. The levels of public good in all cases are quite close to those found with centralized tax policy.

Table 5 Tax and public service policies with differential state taxation

	A $\epsilon_{11} = 0.5,\ \epsilon_{21} = -0.42$			B $\epsilon_{11} = -1.5,\ \epsilon_{21} = 0.42$		
Local public good, efficient	0.1	0.15	0.05	0.1	0.15	0.05
State public good, efficient	0.1	0.05	0.15	0.1	0.05	0.15
Local tax rate	0.336	0.287	0.386	0.162	0.163	0.149
State tax rate (shared)	−0.163	−0.114	−0.213	0.007	0.012	0.028
State tax rate (exclusive)	0.173	0.173	0.173	0.169	0.175	0.169
Shared tax base (k_{ls})	0.257	0.453	0.112	0.520	0.806	0.283
State tax base (k_s)	0.743	0.547	0.888	0.480	0.194	0.717
Local public good	0.075	0.112	0.037	0.071	0.110	0.036
State public good	0.075	0.037	0.112	0.071	0.037	0.107
$\mathrm{MRS}_l = \mathrm{MRS}_s$	1.156	1.156	1.156	1.185	1.185	1.185

4 Concluding comments

Conventional wisdom suggests that separate tax bases for different levels of government are preferred to reduce the extent of vertical externalities and the over-provision of public services associated with the vertical externality. However, many previous studies have not considered the impacts of tax rates across revenues sources, that is, across tax bases. Specifically, how changes in tax rates in one base may influence revenues from other tax bases that are not shared. To the extent that these tax bases are on commodities that are substitutes, a positive, not negative, fiscal externality is generated.

With a strict division of the tax base in which there is no co-occupancy I show that when there are nonzero cross-price elasticities, it is not possible to have both equal tax rates by both governments and equal marginal rates of substitution for their public services, the (second) best solution. That equality in rates and valuation of public services is in general not possible in the absence of co-occupancy means the optimality of co-occupancy cannot be ruled out *a priori*. Here I show that under some conditions, specifically differences in the optimal tax rates in the absence of co-occupancy and underlying cross-price elasticities, co-occupancy might be optimal. In particular, if commodities are substitutes, then co-occupancy is optimal. I also show that if the higher-level government (state) can differentially tax (or subsidize) the shared tax base and the base it alone taxes and can tax the entire base, (second-best) optimality is achieved.

That the optimality of co-occupancy hinges on whether the tax bases of the two levels of government are substitutes or complements suggests the need for empirical examination of cross-base tax effects. While the empirical literature on this issue is limited, there are a few relevant studies. Dahlby and Ferede (2012), for example, find evidence that in Canada, provincial corporate income tax rates increase personal income tax revenues, while provincial sales tax rates reduce them. Bruce et al. (2007) focuses on the determinants of state corporate tax revenues in the USA with their results suggesting that while increases in the top state personal income tax rates lead to modest reductions in corporate income tax revenues, changes in sales tax rates do not seem to have any significant impacts on corporate tax revenues. Finally, Agostini (2014) suggests a negative relationship between state sales tax revenues and both state corporate tax and personal income tax revenues. While the results of these empirical studies are suggestive, it should be noted that they are focusing on cross-base effects at the same level of government, the state or province not cross-base effects at different levels of government, which may be different as the increase in federal taxes should not lead to cross-state or cross-province migration in tax bases. To better understand the vertical fiscal externalities that do exist and their implications for assignment of tax bases across levels of government, further research on these cross-base revenue effects across levels of government is needed.

Here I employ a model in which there are only vertical fiscal externalities and no horizontal externalities. Given empirical evidence (for example, Brülhart and Jametti 2006; Devereux et al. 2007) on the existence of both vertical externalities and horizontal fiscal externalities, how the tax base should be divided among two levels of government when both types of externalities exist is of obvious policy relevance. In a model of cross-border shopping, Hoyt (2016) considers the division of tax base between two levels of government, finding results similar to those found here regarding the efficacy of co-occupancy. However, as Hoyt (2016) allows commodities to differ in the extent of horizontal fiscal externalities associated with their taxation, it also addresses which commodities should be included in the higher-level government (state) tax base and which might be only in the lower-level government tax base.

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