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# Mergers in fiscal federalism☆

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ABSTRACT

We analyze how the merger of regions affects capital tax competition in a two-tier territorial organization where both regions and cities share the same mobile tax base. We identify three effects generated by the merger of regions that impact, either directly or indirectly, both regional and local tax choices: i) an alleviation of tax competition at the regional level, ii) a scale effect in the provision of regional public goods, and iii) a larger internalization of vertical tax externalities generated by cities. We show that the merger of regions always increases regional tax rates while decreasing local tax rates. These results are robust to a change in the timing of the game.



## 1. Introduction

As part of an ongoing process of regionalization in Europe, several European countries have reduced the number of their regions (Dexia Crédit Local, 2008, 2011) with the aim of improving the management of public services. Recent examples include Poland, where the number of "voïvodies" was reduced from 49 to 16 in 1999, and Denmark, where the territorial reform implemented in 2007 replaced the 13 "ämter" with 5 regions. Other countries – including France, Hungary, Italy, Romania and Sweden – are also considering merging regions.

The effect of a merger of same-tier jurisdictions on capital taxation is well-known in a one-tier territorial organization where jurisdictions compete to attract mobile capital. Hoyt (1991) demonstrated that tax rates on mobile capital, and thus public goods provision, increase as the number of jurisdictions decreases. This results from the reduction in the horizontal tax externality: when a jurisdiction increases its tax rate, the capital inflow to other jurisdictions (that become more attractive) is lower. Decreasing the number of jurisdictions reduces the capital movement; thus, increasing the jurisdiction's tax rate is less harmful for that jurisdiction. Considering the possibility of asymmetric mergers, Bucovetsky (2009) also concluded that any merger of two same-tier jurisdictions leads to a higher average tax rate for the federation as a whole due to higher tax rates in jurisdictions that do not belong to the merger.

The effect of a merger of bottom-tier jurisdictions on capital taxation in a two-tier territorial organization with several bottom-tier jurisdictions and a unique top-tier jurisdiction, which share a common mobile tax base, has also been studied. The tax base co-occupation leads to bottom-up vertical tax externalities – in addition to horizontal tax externalities among bottom-tier jurisdictions – since bottom-tier jurisdictions ignore the overall depressive effect that an increase in their tax rate has on the tax base of the unique top-tier jurisdiction (Keen, 1998; Hoyt, 2001; Keen and Kotsogiannis, 2002). With horizontal externalities causing inefficiently low tax rates and vertical externalities causing inefficiently high tax rates, the equilibrium tax rates at the bottom-tier can be either inefficiently low or high. Keen and Kotsogiannis (2004) showed that an increase in the number of bottom-tier jurisdictions unambiguously deteriorates welfare because fiercer tax competition worsens tax externalities. However, the authors were unable to determine whether an increase in the number of bottom-tier jurisdictions would increase or decrease equilibrium tax rates.

Finally, the effect on capital taxation of a "complete merger" of bottom-tier jurisdictions with their top-tier jurisdiction (which is equivalent to removing bottom-tier jurisdictions) in a two-tier territorial organization with several bottom-tier jurisdictions and more than one top-tier jurisdiction, has also been analyzed. Wrede (1997) compared tax choices that result from i) a "competition among

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federations", where $n$ top-tier jurisdictions with several bottom-tier jurisdictions inside each top-tier jurisdiction compete in a Nash game to attract mobile capital, with tax choices that result from ii) a "competition among unitary nations", where only $n$ top-tier jurisdictions compete. He demonstrated that if public goods are substitutes, tax competition among federations leads to less severe under-provision of public goods (or equally higher taxes) than tax competition among unitary nations. Grazzini and Petretto (2007) pursued the analysis in a two-country framework, where one country is federal, consisting of two regions playing as Stackelberg followers with respect to the federal tier, and the other country is unitary. In this asymmetric setting, the co-occupation of the mobile tax base generates horizontal tax externalities between the two countries, in addition to both horizontal externalities at the regional tier and vertical tax externalities in the federal country. By comparing i) the tax game played between a federal structure and a unitary structure with ii) the tax game played between two unitary structures, they showed that the standard "race to the bottom" in the horizontal tax competition literature, according to which two unitary countries competing for attracting mobile capital set inefficiently low tax rates at the equilibrium, can be altered by a change in the institutional setting.

The effect on capital taxation of a merger of top-tier jurisdictions in a two-tier territorial organization with several bottom-tier jurisdictions and several top-tier jurisdictions is, however, unknown. Should one expect an increase in the equilibrium tax rates set by top-tier jurisdictions following the merger, as it would be the case in a one-tier setting (i.e., without bottom-tier jurisdictions)? How does the merger of top-tier jurisdictions affect bottom-tier taxation? What is the consolidated impact for the taxpayer? Our paper addresses these issues.

We consider a two-tier territorial organization with several identical bottom-tier jurisdictions, such as cities, and several identical top-tier jurisdictions, such as regions. Cities and regions tax the same mobile base, that is the amount of capital invested in their territory. Benevolent local and regional governments use their tax revenues in order to finance pure public goods that benefit exclusively their immobile inhabitants. The mobility of the tax base and its co-occupation by both cities and regions generate a two-tier common-pool problem with three types of tax externalities: i) horizontal tax externalities among cities that compete to attract mobile capital, ii) horizontal tax externalities among regions that compete to attract mobile capital and iii) bilateral vertical tax externalities, that is top-down and bottom-up externalities, that arise because tax decisions taken at any tier affect the shared tax base. We thus extend the standard model of capital tax competition among same-tier jurisdictions developed by Zodrow and Mieszkowski (1986) by superimposing an upper tier composed of several top-tier jurisdictions, in contrast to Keen and Kotsogiannis (2002, 2004) who consider a unique top-tier jurisdiction. Wrede (1997) built a similar two-tier tax competition model, with the important difference that his bottom-tier jurisdictions do not take into account the impact of their tax policy on the budget constraint of their top-tier jurisdiction and vice-versa, which rules out major vertical tax externalities.[1]

In view of this elaborate fiscal federalism structure that complicates the capital tax competition model, we must make specific assumptions about citizens' preferences and the production technology to derive a closed form solution. We assume the linearity of the utility function with respect to private, local and regional public goods consumptions, which implies constant marginal rates of substitution between these three types of goods. The decentralized setting allows the coexistence of both local and regional public goods, and the distortionary effect of capital taxation limits the ability to raise tax revenues and therefore ensures the coexistence of both private and public goods. We also assume that the production function is quadratic, so that the demand for capital is a linear function of the interest rate. Relaxing either one or the other of these two assumptions[2] would lead to the emergence of additional effects linked to the merger – in addition to the three effects described below – and complicate the combination of all effects to such an extent that we would no longer be able to sign the impact of the merger.

The impact of an exogenous merger of regions on tax rates is first derived when all jurisdictions, cities and regions, play simultaneously. We identify three effects generated by the merger of regions. The first effect results from the *alleviation of tax competition at the regional level*, which reduces horizontal tax externalities among regions, as shown in the literature (Hoyt, 1991), as well as top-down vertical tax externalities. The merger decreases the number of competing regions, making tax competition at the regional level less fierce, because of a lower capital movement among regions. Regional taxation is less distorsive for regions and – due to tax base-sharing – for their cities, which reduces the incentive to set inefficiently low regional tax rates. The second effect is a *scale effect in the provision of regional public goods*. After the merger, regions (fewer in number) have a larger tax base at the symmetric equilibrium. Therefore, they can provide more pure public good for the same tax rate, which increases the marginal utility from the regional public good provision. Each of these two effects exerts both an upward pressure on regional tax rates and a downward pressure on local tax rates.

The third effect is the *larger internalization of vertical bottom-up tax externalities*. Since cities and regions share the same mobile tax base, a tax increase by a city generates ceteris paribus a capital outflow from the region the city belongs to, because the capital loss in the given city is higher than the sum of capital gain in other cities that belong to the same region. Each city accounts for the impact of a change in its tax rate on the regional tax base, which drives down local tax rates. After the merger, these vertical bottom-up externalities are internalized for a larger regional tax base. Although the capital loss in the city which increases its tax rate is still higher than the sum of capital gain in other cities that belong to the same region, the merger reduces the capital outflow for the region because additional cities with a capital gain join the region. This last effect exerts an upward pressure on local tax rates and a downward pressure on regional tax rates.

The weighted sum of these three effects determines regional and local tax changes following a reduction in the number of regions. We show that the merger of regions always increases regional tax rates while decreasing local tax rates, and that the consolidated tax rate levied on capital (i.e., the sum of regional and local tax rates) is pushed upwards.

The robustness of these results derived in a Nash game is then challenged by altering the timing of the game. Instead of a simultaneous play of regions and cities, we consider two alternative setups: one with regional leadership and the other with local leadership. When regions are Stackelberg leaders, that is, when they anticipate the impact of their own tax decision on the choice of taxation by cities, the merger of regions still exerts an upward pressure on regional tax rates and a downward pressure on local tax rates. However, the consolidated effect is not clear-cut. On the contrary, when cities are Stackelberg leaders, that is, when they anticipate the impact of their own tax decision on the choice of taxation by regions, the outcome of the game is the same as for the simultaneous move Nash game: the strategic advantage of cities is neutralized by the expectation of the action chosen by their region.

[1] Our problem also clearly differs from Wrede (1997), in particular regarding i) the tier concerned by the merger and ii) the nature of the merger. Indeed, we analyze the effect on tax competition of a merger of top-tier jurisdictions instead of a merger of bottom-tier jurisdictions, and our merger does not amount to removing the tier concerned, i.e., there are still some top-tier jurisdictions in our model after the merger whereas the bottom tier disappears in Wrede (1997).

[2] In the tax competition literature, linear preferences are notably assumed by Bucovetsky (2009) and the quadratic function assumption is used by several papers, including Grazzini and van Ypersele (2003), Devereux et al. (2008), Bucovetsky (2009).

Our paper contributes to the significant theoretical literature on tax competition, i) by building a tax competition model in a two-tier framework with more than one top-tier jurisdiction where we remove the critical assumption by Wrede (1997) that governments choose tax rates disregarding the impact on the other tier government, ii) by analyzing how the merger of regions affects distortions linked to tax competition, where we disentangle the three effects at work and iii) by comparing the Nash equilibrium, where cities and regions move simultaneously, and the Stackelberg equilibrium, where either cities or regions move first.

The paper is organized as follows. In Section 2, we present the two-tier tax competition model. In Section 3, we proceed to the analysis of the impact of the merger of regions on the Nash game played by regional and local players. In Section 4, we investigate the consequences of the Stackelberg leadership position. Finally, in Section 5, we provide concluding comments.

## 2. The model

### 2.1. The two-tier territorial organization

Consider a country with two tiers of sub-national jurisdictions, for example, regions and cities. For the sake of simplicity, we assume that the central/federal jurisdiction plays no role. Before the merger, there are $n$ identical regions and, within each region, $m$ identical cities, with $nm$ cities altogether in the country.

The merger of regions is a territorial reorganization where former symmetric regions are broken up to constitute new symmetric regions, which are fewer in number, that is, $\tilde{n} < n$. The merger is exogenously decided; that is, we do not make explicit the forces that lead to a reduction in the number of regions. The number of cities inside each region changes accordingly; that is, it increases from[3] $m$ to $\frac{nm}{\tilde{n}}$. However, the total number $nm$ of cities, and their frontiers do not change. Therefore, after the merger, there are $\tilde{n}$ identical regions and, within each region, $\frac{nm}{\tilde{n}}$ identical cities, with $nm$ cities altogether. Let $i = 1, \ldots, \tilde{n}$ be the index for regions and $j = 1, \ldots, \frac{nm}{\tilde{n}}$ be the index for cities inside each given region.

Pure public goods are provided at both local and regional tiers, with no spillovers and no scale economies.[4] Each local government $ij$ provides a local public good in quantity $g_{ij}$, which is financed by the taxation at a rate $t_{ij}$ of the amount of capital $K_{ij}$ invested in its city. The local budget constraint is thus given by $g_{ij} = t_{ij}K_{ij}$.

Each regional government $i$ provides a regional public good in quantity $G_i$, which is financed by the taxation at a rate $\tau_i$ of the amount of capital $K_i$ employed in region $i$. As each region is composed of $\frac{nm}{\tilde{n}}$ cities after the merger, the regional tax base $K_i$ amounts to the sum of local tax bases inside the region, that is, $K_i \equiv \sum_{j=1}^{nm/\tilde{n}} K_{ij}$. The regional budget constraint is thus given by $G_i = \tau_i \sum_{j=1}^{nm/\tilde{n}} K_{ij}$. Regional and local governments are utilitarian and benevolent. In a strategic game that will be described below, they choose their tax rates to maximize the utility of the representative citizen.

### 2.2. The representative citizen

Citizens are assumed to be identical[5] and immobile. The representative citizen of the city $ij$ derives a utility $v[g_{ij}]$ from the provision of the local public good $g_{ij}$, a utility $V[G_i]$ from the provision of the regional public good $G_i$ and a utility $c_{ij}$ from the consumption of a private good in quantity $c_{ij}$. The utility function of the representative citizen located in $ij$ is thus given by $U[c_{ij}, g_{ij}, G_i] = c_{ij} + v[g_{ij}] + V[G_i]$. Like most papers on capital tax competition, our representative citizen is both the owner of a unique firm located in its city and the owner of an exogenous amount $\bar{k}$ of capital. This amount $\bar{k}$ can be invested in a firm in any city $ij$ to earn a net return on capital, denoted by $\rho_{ij}$, which is equal to the return after local and regional taxes. As we will see afterwards, $\rho_{ij} = \rho\ \forall i, \forall j$ at the symmetric equilibrium. The private consumption $c_{ij}$ thus amounts to the sum of the profit of the firm, denoted by $\prod_{ij}$, and the net remuneration of the capital endowment, such that $c_{ij} = \prod_{ij} + \rho\bar{k}$ at the equilibrium.

Specific assumptions on the functional form of the utility function will be needed to derive explicit solutions for equilibrium tax rates and to sign the impact of the merger. As in Bucovetsky (2009), we assume the linearity of the arguments for both private and public goods consumptions, such that $\frac{\partial^2 U}{\partial c_{ij}^2} = \frac{\partial^2 U}{\partial g_{ij}^2} = \frac{\partial^2 U}{\partial G_i^2} = 0$ where $\frac{\partial^2 U}{\partial g_{ij}^2} \equiv v''[.]$ and $\frac{\partial^2 U}{\partial G_i^2} \equiv V''[.]$. As the first derivative $v'[.]$ (resp. $V'[.]$) is constant, that is the marginal value of an additional dollar of local (resp. regional) tax revenue used to provide the local (resp. regional) public good is a constant, we will subsequently use $v'$ and $V'$ as exogenous parameters to capture marginal utilities.

The marginal utility derived from the local public good provision $g_{ij}$ is proportional to the one derived from the regional public good provision $G_i$, that is, $v' = \alpha V'$, where $\alpha$ is a strictly positive parameter that captures the relative preference for the local public good. In other words, local and regional public goods are perfect substitutes since the marginal rate of substitution is constant. Alternative public policies to reduce emissions of $CO_2$ or to ensure the security of citizens may be examples of public goods that are perfect substitutes. This strong assumption of a linear combination between $g_{ij}$ and $G_i$ makes the coexistence of both local and regional public goods only possible in a decentralized setting, as a central planner would exclusively produce the public good with the highest valuation in a centralized setting, i.e., $g_{ij}$ for $v' > V'$ and $G_i$ for $v' < V'$. In addition, the distortionary effect of capital taxation limits the ability to raise tax revenues and therefore ensures the coexistence of both private and public goods, even if public valuation dominates private valuation. In the case of non-distortionary taxation, the linear combination between private consumption $c_{ij}$ and public goods provision, $g_{ij}$ or $G_i$, would entail the overall taxation of private revenues and, therefore, no private consumption if the public valuation were higher than the private one (i.e., $v' > 1$ at the local tier or $\frac{nm}{\tilde{n}} V' > 1$ at the regional tier). It should be noted that due to the linearity of $V[.]$ w.r.t. $G_i$, the marginal utility derived by citizens from a given amount of regional public good provision is not affected by the merger.

### 2.3. The capital market

The capital market is similar to Wrede (1997). That is, it is a basic one-tier capital market, as modeled by Zodrow and Mieszkowski (1986), on which an additional tier is superimposed. In contrast to Keen and Kotsogiannis (2002, 2004), we have i) a top tier composed of several jurisdictions rather than one and ii) an exogenous supply of capital, as in most models of capital tax competition (Wilson, 1986; Zodrow and Mieszkowski, 1986; Wildasin, 1988).

In each city $ij$, there is a unique firm that is immobile and identical across cities. The firm borrows an amount of capital $K_{ij}$ on the domestic market[6] to produce a composite good in quantity $F[K_{ij}]$ with $F'[.] > 0$ and $F''[.] < 0$. We make the restrictive assumption[7] that

[3] With a view to realism, we assume that $n$, $\tilde{n}$, $m$ and $\frac{nm}{\tilde{n}}$ are natural numbers. In addition, we consider $\tilde{n} > 2$.

[4] Although one of the main reasons why a merger of regions may occur is to exploit scale economies, we abstract from this argument to highlight the pure effect of the merger on tax competition, assuming that the cost of the regional public good provision does not depend on the size of regions.

[5] Admittedly, symmetry is a stark assumption; however, it allows us to simplify our analysis and to rule out any redistributive effects.

[6] The capital market works in autarchy, as both lenders and borrowers reside in the country.

[7] The quadratic assumption is used by several papers on tax competition, including Grazzini and van Ypersele (2003), Devereux et al. (2008), Bucovetsky (2009).

$F'''[.] = 0$ to ensure that the demand for capital is linear, thus enabling us to derive closed form solutions. The profit $\prod_{ij} \equiv F[K_{ij}] - r_{ij}K_{ij}$ is, in its entirety, transferred to the usual owner of the firm in this type of model, that is, the representative citizen. Firm profit maximizing behavior implies the familiar condition of remuneration at the marginal productivity of capital, $F'[K_{ij}] = r_{ij}\ \forall i, \forall j$. The resulting demand for capital $K_{ij}[r_{ij}]$ and profit $\prod_{ij}[r_{ij}]$ are decreasing functions of the interest rate $r_{ij}$, that is, $K'_{ij}[r_{ij}] = \frac{1}{F''} < 0$ and $\Pi'_{ij}[r_{ij}] = -K_{ij} < 0\ \forall i, \forall j$.

For each unit of capital invested by a capital owner in the firm located in city $ij$, two source-based tax rates are levied: a tax rate $t_{ij}$ levied by the city $ij$ and a tax rate $\tau_i$ levied by the region $i$. The assumption of tax-base sharing, i.e., the fact that both local and regional jurisdictions independently tax the same mobile tax base, is classic in the literature on vertical capital tax competition (Keen, 1998; Keen and Kotsogiannis, 2002). It is also consistent with empirical evidence; for example, both American states and local jurisdictions levy taxes on mobile business capital (see Fox et al., 2010, who empirically tested vertical reaction functions for capital taxes in this country).

Capital is assumed to be mobile without cost in a perfectly competitive market. It moves across cities and, thus, across regions until it earns the same net return $\rho$ everywhere, that is, $\rho = r_{ij} - (t_{ij} + \tau_i)\ \forall i, \forall j$.

The national supply of capital is the sum of the initial endowment $\overline{k}$ of the $nm$ representative citizens of the country, i.e., $nm\overline{k}$. Given that $r_{ij} = \rho + \tau_i + t_{ij}\ \forall i, \forall j$, the capital market-clearing condition $\sum_{i=1}^{\tilde{n}}\sum_{j=1}^{nm/\tilde{n}} K_{ij}[\rho + \tau_i + t_{ij}] = nm\overline{k}$ implicitly defines the equilibrium value of the net return on capital, $\rho(\boldsymbol{\tau}, \mathbf{t}_1, \ldots, \mathbf{t_i}, \ldots, \mathbf{t}_{\tilde{n}})$ with $\boldsymbol{\tau} = (\tau_1, \ldots \tau_{\tilde{n}})$ and $\mathbf{t_i} = \left(t_{i1}, \ldots, t_{ij}, \ldots, t_{i\frac{nm}{\tilde{n}}}\right) \forall i$. Differentiating the market-clearing condition yields, at the symmetric equilibrium:

$$\frac{\partial \rho}{\partial \tau_i} = \frac{-\sum_{j=1}^{nm/\tilde{n}} K'_{ij}}{\sum_{i=1}^{\tilde{n}}\sum_{j=1}^{nm/\tilde{n}} K'_{ij}} = \frac{-1}{\tilde{n}}, \quad \frac{\partial r_{ij}}{\partial \tau_i} = 1 + \frac{\partial \rho}{\partial \tau_i} = \frac{\tilde{n}-1}{\tilde{n}}, \quad \frac{\partial r_{ij}}{\partial \tau_{-i}} = \frac{\partial \rho}{\partial \tau_{-i}} = \frac{-1}{\tilde{n}},$$

$$\frac{\partial \rho}{\partial t_{ij}} = \frac{-K'_{ij}}{\sum_{i=1}^{\tilde{n}}\sum_{j=1}^{nm/\tilde{n}} K'_{ij}} = \frac{-1}{nm}, \quad \frac{\partial r_{ij}}{\partial t_{ij}} = 1 + \frac{\partial \rho}{\partial t_{ij}} = \frac{nm-1}{nm},$$

$$\frac{\partial r_{ij}}{\partial t_{-ij}} = \frac{\partial r_{ij}}{\partial t_{i,-j}} = \frac{\partial \rho}{\partial t_{-ij}} = \frac{\partial \rho}{\partial t_{i,-j}} = \frac{-1}{nm}.$$

The mobility of the tax base and its co-occupation by both cities and regions generate a two-tier common-pool problem. Regional and local tax choices both affect the location choice of capital since the net return on capital decreases when the cumulative tax rate $\tau_i + t_{ij}\ \forall i, \forall j$ increases. Therefore, three types of tax externalities emerge in our two-tier setting:

i) *horizontal externalities at the local level*. Since capital is perfectly mobile, each city individually has an incentive to reduce its tax rate on capital to attract a larger tax base, i.e., $-K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}} > 0$, which generates a negative externality towards the other cities by shrinking their tax base, i.e., $-K'_{lk}\frac{\partial \rho}{\partial t_{ij}} < 0\ \forall l \neq i, \forall k$ and $-K'_{ik}\frac{\partial \rho}{\partial t_{ij}} < 0\ \forall k \neq j$. Thus, the other cities must reduce their tax rate to retain their tax base. This strategic behavior leads to a downward spiral in tax rates, usually called the "race to the bottom", as each city tries to undercut the others by setting an attractive tax rate. Equilibrium tax rates are, therefore, lower than those that would be chosen under coordination.

ii) *horizontal externalities at the regional level*. Each region similarly undercuts the other regions via the choice of its tax rate. The strategic choice of reducing its tax rate $\tau_i$ expands the region $i$'s tax base, i.e., $-\sum_{j=1}^{nm/\tilde{n}} K'_{ij}\frac{\partial r_{ij}}{\partial \tau_i} > 0$ but hurts the other regions, i.e., $-\sum_{j=1}^{nm/\tilde{n}} K'_{lj}\frac{\partial \rho}{\partial \tau_i} < 0\ \forall l \neq i$. However, regional tax competition is less fierce than local tax competition because the number of competitors is lower, thus, implying that regional tax externalities are weaker. This is captured by the fact that the impact on the net return on capital of the regional tax rate is lower than the one of the local tax rate, i.e., $\frac{\partial \rho}{\partial t_{ij}} > \frac{\partial \rho}{\partial \tau_i}$.

iii) *vertical externalities between local and regional tiers*. Cities and regions independently tax the same mobile tax base, which generates both bottom-up externalities (as the tax choice of a city affects the ñ regions) and top-down tax externalities[8] (as the tax choice of a region affects the nm cities). Suppose first that the city $ij$ reduces its tax rate $t_{ij}$. Ceteris paribus, capital will leave other cities in region $i$ as well as cities in other regions, to locate in city $ij$, thus creating an overall positive impact on region $i$'s tax base,[9] i.e., $-\sum_{k \neq j}^{nm/\tilde{n}} K'_{ik}\frac{\partial \rho}{\partial t_{ij}} - K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}} > 0$, and a negative impact on other regions $l \neq i$, i.e., $-\sum_{j=1}^{nm/\tilde{n}} K'_{lj}\frac{\partial \rho}{\partial t_{ij}} < 0$, at the symmetric equilibrium ($K'_{ik} = K'_{ij} < 0$ for all $i, k, j$). Suppose now that the region $i$ reduces its tax rate $\tau_i$. Ceteris paribus, capital will leave cities that belong to other regions ($l \neq i$) to locate in the $\frac{nm}{\tilde{n}}$ cities that belong to region $i$. The capital inflow for region $i$ is $-\sum_{j=1}^{nm/\tilde{n}} K'_{ij}\frac{\partial r_{ij}}{\partial \tau_i} > 0$. Therefore, this creates positive vertical top-down externalities for each city $ij$ inside region $i$, i.e., $-K'_{ij}\frac{\partial r_{ij}}{\partial \tau_i} > 0\ \forall i, j$, but also negative vertical top-down externalities for each city $lj$ of other regions, i.e., $-K'_{lj}\frac{\partial \rho}{\partial \tau_i} < 0\ \forall l \neq i, \forall j$.

To summarize, the choice of a jurisdiction's tax rate affects its own tax base, the tax base of other same-tier jurisdictions and the tax base of other-tier jurisdictions, because of the combination of tax-base sharing and mobility.

Wrede (1997) assumes that "each government takes only its own budget restriction into consideration", although each tier maximizes the utility of its representative citizen, which depends on both bottom-tier and top-tier public goods provisions (as in our model). Therefore, the bottom-tier does not take into account the impact of its tax choice on the provision of top-tier public goods, and vice-versa. A major difference with Wrede is that we depart from this simplification, which has important consequences in terms of internalization of tax externalities. As shown later, each city will internalize part of the bottom-up tax externalities, i.e., it will take into account the impact of a change in its local tax rate on the public goods provided by its region, but not by other regions. Additionally, each region will internalize part of the top-down tax externalities, i.e., it will take into account the impact of a change in its regional tax rate on the public goods provided by cities located in its region, but not by cities located in other regions. Thus, the internalization of vertical tax externalities, whether bottom-up or top-down, will be partial.

Finally, let $\varepsilon_{\tau_i} = \frac{\partial\left(\sum_{j=1}^{nm/\tilde{n}} K_{ij}\right)}{\partial \tau_i}\frac{\tau_i}{\sum_{j=1}^{nm/\tilde{n}} K_{ij}} < 0$ denote the elasticity of capital invested in region $i$ with respect to region $i$'s tax rate and $\varepsilon_{t_{ij}} =$

[8] In Keen and Kotsogiannis (2002, 2004), there are only bottom-up vertical tax externalities, due to the fact that the public good is a publicly provided private good. The absence of top-down vertical tax externalities is explained by the fact that the utility function of the unique top-tier government is the sum of the utility of all bottom-tier jurisdictions and therefore the top-tier jurisdiction perfectly internalizes top-down externalities. This is no longer the case with several top-tier jurisdictions.

[9] At the symmetric equilibrium, $K'_{ik} = K'_{ij} < 0$ for all $i, k, j$, so $-\sum_{k \neq j}^{nm/\tilde{n}} K'_{ik}\frac{\partial \rho}{\partial t_{ij}} - K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}} = -\left(\frac{nm}{\tilde{n}} - 1\right)K'_{ik}\frac{\partial \rho}{\partial t_{ij}} - K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}} = -K'_{ij}\left(\frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ij}} + 1\right) > 0$.

$\frac{\partial K_{ij}}{\partial t_{ij}}\frac{t_{ij}}{K_{ij}}<0$ denote the elasticity of capital invested in city $ij$ with respect to city $ij$'s tax rate. A change in the tax rate $t_{ij}$ (resp. $\tau_i$) produces two opposite effects on tax revenues: (i) a direct positive effect $K_{ij}dt_{ij}$ (resp. $\sum_{j=1}^{nm/\tilde{n}} K_{ij}d\tau_i$) and (ii) an indirect negative effect through the net return on capital $\varepsilon_{t_{ij}}K_{ij}dt_{ij}$ (resp. $\varepsilon_{\tau_i}\sum_{j=1}^{nm/\tilde{n}} K_{ij}d\tau_i$). Consistent with empirical findings,[10] we postulate that elasticities belong to the interval]−1, 0[, which implies $\frac{\partial\left(\tau_i\sum_{j=1}^{nm/\tilde{n}}K_{ij}\right)}{\partial\tau_i} = (1+\varepsilon_{\tau_i})\sum_{j=1}^{nm/\tilde{n}} K_{ij} > 0$ and $\frac{\partial(t_{ij}K_{ij})}{\partial t_{ij}} = \left(1+\varepsilon_{t_{ij}}\right)K_{ij} > 0$. Therefore, tax revenues of a jurisdiction always increase when the jurisdiction's tax rate increases. Because our governments are benevolent, these additional tax revenues are entirely used to produce more public goods.

### 2.4. The impact of the merger on the capital market

We first note that the impact of local taxation on the net return, $\frac{\partial\rho}{\partial t_{ij}}\,\forall i, \forall j$, remains the same whatever the number of regions, $\tilde{n}$, as the total number of cities, $nm$, does not change. The merger of regions, therefore, has no impact on the fierceness of horizontal tax competition at the local level, i.e., $\frac{\partial}{\partial\tilde{n}}\left(\frac{\partial\rho}{\partial t_{ij}}\right) = \frac{\partial}{\partial\tilde{n}}\left(\frac{\partial r_{ij}}{\partial t_{ij}}\right) = 0$. Due to symmetry and a fixed supply of capital, we will show that the allocation of capital among cities does not change.

In contrast, the merger of regions reduces the impact of an increase in the regional tax rate on both the net return on capital and the interest rate, i.e., $\frac{\partial}{\partial\tilde{n}}\left(\frac{\partial\rho}{\partial\tau_i}\right) = \frac{\partial}{\partial\tilde{n}}\left(\frac{\partial r_{ij}}{\partial\tau_i}\right) = \frac{1}{\tilde{n}^2}$. Horizontal tax competition for capital at the regional level becomes less fierce. In other words, the market share of each region, which is equal to the inverse of the number of regions, increases with the merger. It should be noted that $\frac{\partial\rho}{\partial\tau_i} = -1$ and $\frac{\partial r_{ij}}{\partial\tau_i} = 0$ when capital is completely inelastic or without regional tax competition for capital.

The merger of regions also increases each regional tax base since the fixed national supply of capital $nm\bar{k}$ is equally divided among fewer regions at the symmetric equilibrium, i.e., $\sum_{j=1}^{nm/\tilde{n}} K_{ij} > \sum_{j=1}^{m} K_{ij}$.

### 2.5. The timing of the game

Regional and local governments play a Nash game. Regional governments simultaneously select their tax rate to maximize the welfare of the representative citizen residing within their region, taking as given tax rates chosen by the other regions and cities. Simultaneously, local governments select their tax rate to maximize the welfare of the representative citizen residing within their city, taking as given tax rates chosen by the other cities and regions.

Regional and local public goods are determined as residuals after taxes are collected. Firms then choose the amount of capital that maximizes their profits given these tax policies and production takes place. Finally, profits are distributed, and citizens enjoy the consumption of both private and public goods. These last two stages are implicitly introduced in our analysis: regional and local governments take into account the reaction of the capital demand when choosing their tax strategy,[11] and as regional and local governments are benevolent, citizens' preferences guide the choices of governments.

The robustness of the results derived in the Nash game will then be challenged by altering the timing of the game. Instead of a simultaneous play of regions and cities, we will consider two alternative setups:

i) *a Stackelberg game where regional governments move first*. Regional leaders move first and then local followers move sequentially. In being the first to choose their tax rate, regional governments possess a commitment power.
ii) *a Stackelberg game where local governments move first*. Local leaders move first, choosing their best tax strategy. Regional followers then observe local choices and select their tax rate.

## 3. Mergers in a simultaneous Nash game

All jurisdictions, cities and regions, simultaneously choose their tax rates, given the strategies of the other players. After solving the regional government's problem and the local government's problem, we will determine the impact on tax rates of a reduction in the number of regions using comparative statics.

### 3.1. The regional government's problem

Each regional government $i$ for $i = 1,\ldots,\tilde{n}$ chooses the tax rate $\tau_i$, which maximizes the utility of the representative citizen located in its region, taking as given the tax choices of other regions and cities. It thus solves the problem:

$$\underset{\tau_i}{Max}\quad \sum_{j=1}^{nm/\tilde{n}}\left(c_{ij} + v\left[g_{ij}\right] + V[G_i]\right)$$
$$s.t.\quad c_{ij} = \Pi_{ij}\left[r_{ij}\right] + \rho\bar{k},\quad g_{ij} = t_{ij}K_{ij}\left[r_{ij}\right]\ \text{and}\ G_i = \tau_i\sum_{j=1}^{nm/\tilde{n}} K_{ij}\left[r_{ij}\right].$$

Imposing a condition that ensures the concavity of the problem faced by the regional government (see Appendix A), the first-order condition[12]:

$$\sum_{j=1}^{nm/\tilde{n}}\left(\Pi'_{ij}\frac{\partial r_{ij}}{\partial\tau_i} + \frac{\partial\rho}{\partial\tau_i}\bar{k} + v' t_{ij}K'_{ij}\frac{\partial r_{ij}}{\partial\tau_i} + V'\left(1+\varepsilon_{\tau_i}\right)\sum_{j=1}^{nm/\tilde{n}} K_{ij}\right) = 0 \qquad (1)$$

determines the regional government's reaction function $\{\tau_i(\mathbf{t}_1,\ldots,\mathbf{t}_i,\ldots,\mathbf{t}_{\tilde{n}};\boldsymbol{\tau}_{-i})\}_i$, which depends on the vector $\mathbf{t}_i = \left(t_{i1},\ldots,t_{i\frac{nm}{\tilde{n}}}\right)$ of local tax rates in each region $i = 1, \ldots, \tilde{n}$ and on the vector $\boldsymbol{\tau}_{-i} = \left(\tau_1,\ldots,\tau_{i-1},\tau_{i+1},\ldots,\tau_{\tilde{n}}\right)$ of other regional tax rates. According to Eq. (1), each region $i$ determines its tax rate to equalize the marginal costs of a reduction in private consumption and local public good provision, that is, $\frac{\partial c_{ij}}{\partial\tau_i} + v'\frac{\partial g_{ij}}{\partial\tau_i}<0$, and the marginal benefit of a rise in regional public good provision, that is, $V'\frac{\partial G_i}{\partial\tau_i} = V'\left(1+\varepsilon_{\tau_i}\right)\sum_{j=1}^{nm/\tilde{n}} K_{ij} > 0$, following an increase in $\tau_i$.

### 3.2. The local government's problem

Simultaneously, each local government $ij$ for $i = 1, \ldots, \tilde{n}$ and $j = 1,\ldots,\frac{nm}{\tilde{n}}$ chooses the tax rate $t_{ij}$, which maximizes the utility of the representative citizen located in its city, taking as given the tax choices of other cities and regions. It thus solves the problem:

$$\underset{t_{ij}}{Max}\quad c_{ij} + v\left[g_{ij}\right] + V[G_i]$$
$$s.t.\quad c_{ij} = \Pi_{ij}\left[r_{ij}\right] + \rho\bar{k},\quad g_{ij} = t_{ij}K_{ij}\left[r_{ij}\right]\ \text{and}\ G_i = \tau_i\sum_{j=1}^{nm/\tilde{n}} K_{ij}\left[r_{ij}\right]$$

[10] See Chirinko et al. (1999) for instance.

[11] The strategic variable is the tax rate because our aim is to analyze the impact of mergers on tax competition. However, in line with Wildasin (1988), we could show that Nash (resp. Stackelberg) equilibria in which tax rates are the strategic variables do not coincide with Nash (resp. Stackelberg) equilibria in which the amount of public good is the strategic variable, because the public good provision does not affect the location of capital.

[12] At the symmetric equilibrium, distortive effects – through the net return on capital – on private consumption compensate each other, i.e., $\Pi'_{ij}\frac{\partial\rho}{\partial\tau_i} + \frac{\partial\rho}{\partial\tau_i}\bar{k} = 0$ since $\Pi'_{ij} = -K_{ij}$ and $K_{ij} = \bar{k}\ \forall i, \forall j$, implying that $\frac{\partial c_{ij}}{\partial\tau_i} = \Pi'_{ij}<0$.

Imposing a condition that ensures the concavity of the problem faced by the local government (see Appendix A), the first-order condition,[13]

$$\Pi'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}}+\frac{\partial \rho}{\partial t_{ij}}\overline{k}+v'\left(1+\varepsilon_{t_{ij}}\right)K_{ij}+V'\tau_i\left(\sum_{k\neq j}K'_{ik}\frac{\partial \rho}{\partial t_{ij}}+K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}}\right)=0 \qquad (2)$$

determines the local government's reaction function $\left\{t_{ij}(\boldsymbol{\tau};\mathbf{t_1},\ldots,\mathbf{t_{i,-j}},\ldots,\mathbf{t_{\tilde{n}}})\right\}_{ij}$, which depends on the vector $\mathbf{t_k}=\left(t_{k1},\ldots,t_{k\frac{nm}{\tilde{n}}}\right)$ of local tax rates in each region $k=1,\ldots,i-1,i+1,\ldots,\tilde{n}$, on the vector $\mathbf{t_{i,-j}}=\left(t_{i1},\ldots,t_{ij-1},t_{ij+1},\ldots,t_{i\frac{nm}{\tilde{n}}}\right)$ of tax rates of cities other than $ij$ in region $i$ and on the vector $\boldsymbol{\tau}=(\tau_1,\ldots,\tau_i,\ldots,\tau_{\tilde{n}})$ of regional tax rates. The tax rate chosen by the local government $ij$ is such that it equalizes the marginal costs of a reduction in private consumption and regional public good provision, that is, $\frac{\partial c_{ij}}{\partial t_{ij}}+V'\frac{\partial G_i}{\partial t_{ij}}<0$, and the marginal benefit of a rise in local public good provision, that is, $v'\frac{\partial g_{ij}}{\partial t_{ij}}=v'\left(1+\varepsilon_{t_{ij}}\right)K_{ij}>0$, following an increase in $t_{ij}$.

Solving first-order conditions (1) and (2) for all cities and regions simultaneously determines the Nash equilibrium levels of tax rates. In the remainder of the paper, results will be derived provided that a set of conditions specified in Appendix A are satisfied. Condition (5) derived from the regional SOC, i.e., $2\frac{nm}{\tilde{n}}V'\geq\frac{\partial r_{ij}}{\partial \tau_i}$, and condition (6) derived from the local SOC, i.e., $2v'\geq\frac{\partial r_{ij}}{\partial t_{ij}}$, ensure the concavity of regional and local objective functions and the uniqueness of the solution. Marginal utilities from public goods must be high enough to avoid a corner solution. Condition (8), i.e., $\left(\frac{nm}{\tilde{n}}V'-1\right)\left(\frac{nm-1}{nm}/\frac{\tilde{n}-1}{\tilde{n}}\right)\geq(v'-1)$, guarantees that regional tax rates chosen at the symmetric equilibrium are positive and condition (9), i.e., $(v'-1)\geq\left(\frac{nm}{\tilde{n}}V'-1\right)\frac{\tilde{n}}{nm}$, guarantees that local tax rates chosen at the symmetric equilibrium are positive. These two conditions of positivity determine a non-empty range of parameters $\alpha$, which ensures that both local and regional tax rates are positive. Beyond this interval, either local or regional public goods will not be produced. Condition (9) states that the relative preference for the local public good $\alpha=\frac{v'}{V'}$ must be sufficiently high to ensure that local public goods are produced. Similarly, condition (8) states that the relative preference for the local public good $\alpha$ must not exceed a threshold value beyond which regional public goods would no longer be produced. Indeed, for high values of $v'$ relative to $V'$, the marginal cost of a reduction in the local public good provision may exceed the marginal benefit from a higher provision of the regional public good, following a rise in $\tau_i$; therefore, the regional tax rate $\tau_i$ will be set to 0. In addition, as shown in Lemma 2 in Appendix A, the conditions of positivity require that the marginal utility derived from the provision of the public good (whether local or regional) exceeds the marginal utility derived from the provision of the private good (normalized to unity), i.e., $v'>1$ and $\frac{nm}{\tilde{n}}V'>1$. If this were not the case, governments would not levy tax revenues to finance public goods.

### 3.3. Implications of the merger of regions

We now use the comparative statics to examine in more detail the impact of the merger of regions on regional and local tax rates. Differentiating the first-order conditions (1) and (2) with respect to $\tau_i$, $t_{ij}$ and $\tilde{n}$ and using Cramer's rule, we have:

$$\frac{\partial \tau_i}{\partial \tilde{n}}=\frac{1}{A^N}((E1+E2)*HORIREG-E3*VERTI) \qquad (3)$$

and

$$\frac{\partial t_{ij}}{\partial \tilde{n}}=\frac{1}{A^N}(-(E1+E2)*VERTI+E3*HORILOC) \qquad (4)$$

where the expressions $A^N$, $E1$, $E2$, $E3$, $HORIREG$, $HORILOC$ and $VERTI$ are defined in Appendix B. Since $A^N>0$ (see Lemma 3 in Appendix B), the impact of the merger of regions on regional tax rates, i.e., $\frac{\partial \tau_i}{\partial \tilde{n}}$, and the impact of the merger of regions on local tax rates, i.e., $\frac{\partial t_{ij}}{\partial \tilde{n}}$, depend on the interplay of three effects, $E1$, $E2$ and $E3$, which are transmitted either horizontally (at a weight $HORIREG$ or $HORILOC$) or vertically (at a weight $VERTI$).

The first effect, $E1\equiv-\left(v't_{ij}+V'\frac{nm}{\tilde{n}}\tau_i\right)K'_{ij}\frac{\partial(\partial\rho/\partial\tau_i)}{\partial\tilde{n}}>0$, results from the *alleviation of horizontal tax competition at the regional level*. By reducing the number of competing regions, the merger makes tax competition at the regional level less fierce due to lower capital movement. A region $i$'s tax increase generates less capital outflow from region $i$ (and therefore less capital inflow to other regions) since the marginal impact $\frac{\partial\rho}{\partial\tau_i}$ of the regional tax rate on the equilibrium value of the net return on capital $\rho$ becomes lower, i.e., $-\frac{\partial(\partial\rho/\partial\tau_i)}{\partial\tilde{n}}<0$. The merger thus reduces horizontal tax externalities at the regional level, as shown by Hoyt (1991). Because of tax-base sharing, it also reduces vertical top-down tax externalities: a region $i$'s tax increase generates less capital outflow from cities $ij$ $\forall j$ located in its territory and less capital inflow to cities located in other regions. The reduction in horizontal tax externalities increases the marginal utility derived from the regional public good $\left(-V'\frac{nm}{\tilde{n}}\tau_iK'_{ij}\frac{\partial(\partial r_{ij}/\partial\tau_i)}{\partial\tilde{n}}>0\right)$ and the reduction in vertical top-down tax externalities increases the marginal utility derived from the local public good $\left(-v't_{ij}K'_{ij}\frac{\partial(\partial r_{ij}/\partial\tau_i)}{\partial\tilde{n}}>0\right)$ without changing the cost in terms of private consumption, thus lowering the incentive for regional governments to set inefficiently low tax rates.

The second effect, $E2\equiv-V'\frac{\partial(nm/\tilde{n})}{\partial\tilde{n}}(1+\varepsilon_{\tau_i})K_{ij}>0$, is a *scale effect in the provision of regional public goods*. After the merger, regions (fewer in number) have larger tax bases at the symmetric equilibrium. More tax revenues can be collected and thus a greater amount of regional public good can be provided after the merger, for the same tax rate. The marginal utility derived from a regional tax increase becomes higher – without changing the cost in terms of private consumption – thus pushing up the regional tax rate. This effect is strongly linked to our assumption that the regional good is a pure public good, the cost of provision of the regional public good being the same whatever the size of regions (or, equally, whatever the number of public good "consumers" in each region). In case of a publicly provided private good at the regional level, $E2$ would disappear without altering the whole impact $\frac{\partial\tau_i}{\partial\tilde{n}}$ of the merger on the regional tax rate, as shown in the bullet point "Sign of $\frac{\partial\tau_i^N}{\partial\tilde{n}}$" in Appendix C.

The third effect, $E3\equiv-V'\tau_i\frac{\partial}{\partial\tilde{n}}\left(\sum_{k\neq j}K'_{ik}\frac{\partial\rho}{\partial t_{ij}}+K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}}\right)=-V'\tau_iK'_{ij}\frac{\partial\left(1+\frac{nm}{\tilde{n}}\frac{\partial\rho}{\partial t_{ij}}\right)}{\partial\tilde{n}}>0$, captures the *larger internalization of vertical bottom-up tax externalities*. Each city $ij$ neglects the impact its tax choice $t_{ij}$ has on the provision of local public goods in other cities (whether they belong to the same region or not), i.e. it does not internalize horizontal externalities at the local tier. However each city $ij$ accounts for the impact of its tax choice $t_{ij}$ on the regional public good provided inside its region, i.e., it internalizes vertical bottom-up externalities for its region (but not for other regions). As described before, if city $ij$ increases its tax rate $t_{ij}$, this generates a capital outflow from city $ij$ as well as a capital outflow from region $i$, because the capital outflow from city $ij$ partially relocates in cities outside of region $i$. City $ij$ accounts for this capital movement out of region $i$ when choosing its tax rate, which drives down local tax rates. Due to symmetry in our model, the capital outflow from city $ij$ is equally distributed among all the other cities of the

[13] At the symmetric equilibrium, distortive effects – through the net return on capital – on private consumption compensate each other, implying that $\frac{\partial c_{ij}}{\partial t_{ij}}=\Pi'_{ij}<0$. Furthermore, marginal demands for capital are identical, i.e., $K'_{ik}=K'_{ij}\forall i,j,k$, implying that $\frac{\partial G_i}{\partial t_{ij}}=\tau_i\left((m-1)K'_{ij}\frac{\partial\rho}{\partial t_{ij}}+K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}}\right)=\tau_i\left(1+m\frac{\partial\rho}{\partial t_{ij}}\right)K'_{ij}$.

country, which implies that the amount of capital outflow from region $i$ is in inverse proportion to the number of cities inside region $i$. The larger region $i$ is, the lower the capital outflow from region $i$ (as more cities are located inside of region $i$), and as a result, the lower the incentive for each city $ij$ to reduce its tax rate. By increasing the size of regions, the merger thus leads each city to internalize vertical bottom-up externalities for a larger regional tax base. In the extreme case where all regions merge to form a unique top-tier jurisdiction, vertical bottom-up tax externalities would be entirely internalized. Note that for $K'_{ij} = 0\,\forall j$, capital becomes inelastic to a change in the gross return on capital, and all of these effects vanish.

The first two effects, $E1$ and $E2$, are considered as regional because they originate from the regional tier, whereas the third one, $E3$, is considered as local because it originates from the local tier.[14] Due to the two-tier territorial organization, these three effects are both horizontally transmitted, i.e., at the regional (resp. local) tier if they are regional (resp. local) at a weight *HORIREG* (resp. *HORILOC*), and vertically transmitted, i.e., at the regional (resp. local) tier if they are local (resp. regional) at a weight *VERTI*.[15]

Each effect taken individually favors an increase in tax pressure when it is horizontally transmitted*: the alleviation of horizontal tax competition at the regional level* ($E1$) and the *scale effect in the provision of regional public goods* ($E2$) both drive regional tax rates upwards, whereas the *larger internalization of vertical bottom-up tax externalities* ($E3$) yields higher local tax rates following a decrease in $\tilde{n}$. However, due to the overlapping structure, each effect is also vertically transmitted, either top-down (for $E1$ and $E2$) or bottom-up ($E3$), where it favors a decrease in tax pressure: the *alleviation of horizontal tax competition at the regional level* ($E1$) and the *scale effect in the provision of regional public goods* ($E2$) both tend to reduce local tax rates, whereas the *larger internalization of vertical bottom-up tax externalities* ($E3$) encourages regions to lower their tax rate following the merger of regions.

The overall impact of the merger on tax rates, $\frac{\partial \tau_i}{\partial \tilde{n}}$ and $\frac{\partial t_{ij}}{\partial \tilde{n}}$, is a weighted sum of these three effects, which are transmitted both horizontally and vertically. By summing $\frac{\partial \tau_i}{\partial \tilde{n}}$ and $\frac{\partial t_{ij}}{\partial \tilde{n}}$, we find the impact of the merger of regions on the consolidated tax rate $T_{ij} \equiv \tau_i + t_{ij}$. It follows that:

**Proposition 1.** *In a Nash game, the merger of regions always increases regional tax rates and decreases local tax rates, with an overall increasing impact on the consolidated tax rate.*

**Proof.** See Appendix C

The merger of regions has an unambiguous impact on both regional and local tax rates. Following the territorial reorganization, regions, fewer in number, increase their tax rates, whereas cities reduce their tax rates. We learn from Proposition 1 that the weighted sum of the *alleviation of horizontal tax competition at the regional level* and the *scale effect in the provision of regional public goods*, i.e., $E1$ and $E2$ (which are transmitted at a weight *HORIREG* at the regional level and *VERTI* at the local level), is higher than the *larger internalization of vertical bottom-up tax externalities*, i.e., $E3$ (which is transmitted at a weight *VERTI* at the regional level and *HORILOC* at the local level).

If the linear utility assumption were relaxed, the three effects $E1$, $E2$, $E3$ would still exist. Assuming strictly concave functions v(.) and V(.), two new effects $E4 \equiv -V''\tau_i \frac{\partial(nm/\tilde{n})}{\partial \tilde{n}} K_{ij}(1+\varepsilon_{\tau_i})\sum_j K_{ij}$ and $E5 \equiv -V''\tau_i \frac{\partial(nm/\tilde{n})}{\partial \tilde{n}} K_{ij}\tau_i\left(\sum_{k\neq j} K'_{ik}\frac{\partial \rho}{\partial t_{ij}} + K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}}\right)$ would appear and the three weights *HORIREG*, *HORILOC* and *VERTI* would be larger. However, new conditions of concavity and the inability to derive equilibrium tax rates would prevent us from signing $\frac{\partial \tau_i}{\partial \tilde{n}}$ and $\frac{\partial t_{ij}}{\partial \tilde{n}}$ without introducing a new set of strong ad-hoc assumptions. Similar statements can be made concerning the quadratic production function assumption.

In the case of coalitions of regions[16] that coordinate to jointly choose their tax rate, the territorial organization would not change; that is, the number of regions and the number of cities inside each region would remain stable. The three effects $E1$, $E2$, $E3$ would thus be absent. In particular, the impact of regional taxation on the net return would not change after the coalition of regions, i.e., $\frac{\partial \rho}{\partial \tau_i} = \frac{1}{n}$.

## 4. Mergers in a Stackelberg game

We now test whether our results are robust to a change in the timing of the game by considering two alternative sequences of decisions. First, we assume that regional governments act as Stackelberg leaders vis-à-vis local governments. The larger size of higher-tier jurisdictions may be a reason to justify regional leadership (see Keen and Kotsogiannis, 2003, for instance). Second, we assume that local governments act as Stackelberg leaders vis-à-vis regional governments. A recent literature on the soft budget constraint issue applied to fiscal federalism uses local leadership to capture the weakness of the higher-tier in terms of intergovernmental transfers (Vigneault, 2007; Breuillé and Vigneault, 2010).

The leader moves first and chooses the tax rate that maximizes the welfare over its territory, anticipating the predicted response of the follower. The follower then observes the decision of the leader and in equilibrium selects its tax rate as a response.

### 4.1. Regional governments are Stackelberg leaders

When regional governments are Stackelberg leaders, they anticipate the impact of their own tax decision on the choice of taxation by local governments. Best response functions $\hat{t}_{ij}(\tau_i)$ of local followers are calculated by differentiating first-order conditions (Eq. (2)) by $t_{ij}$ and $\tau_i$ for all $i = 1, \ldots, \tilde{n}$ and $j = 1, \ldots, \frac{nm}{\tilde{n}}$. At the symmetric equilibrium, we obtain[17]:

$$\frac{\partial \hat{t}_{ij}}{\partial \tau_i} = -\frac{\left(v'\frac{\partial r_{ij}}{\partial \tau_i} + V'\left(1+\frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right) - \frac{\partial r_{ij}}{\partial t_{ij}}\frac{\partial r_{ij}}{\partial \tau_i}\right)}{\left(2v' - \frac{\partial r_{ij}}{\partial t_{ij}}\right)\frac{\partial r_{ij}}{\partial t_{ij}}} \in ]-1;0[ \quad \forall i, \forall j.$$

As expected,[18] local reaction functions are negative. Local and regional tax rates are therefore strategic substitutes, which implies that all local governments $j$ located in region $i$ will respond to a rise in $\tau_i$ by reducing their tax rates $t_{ij}$ to restore some competitiveness. It should be noted that $\frac{\partial(\partial \hat{t}_{ij}/\partial \tau_i)}{\partial \tilde{n}} < 0$, that is, the higher the number of regions, the more responsive the local tax rate to a change in the regional tax rate.

Regional leaders select the tax rate that maximizes the utility of the representative citizen located in their region, anticipating the predicted response of local followers, i.e.,

$$\underset{\tau_i}{Max} \sum_{j=1}^{nm/\tilde{n}} \left(c_{ij} + v\left[g_{ij}\right] + V[G_i]\right)$$

$$s.t. \quad c_{ij} = \Pi_{ij}\left[r_{ij}\right] + \rho\bar{k}, \quad g_{ij} = t_{ij}K_{ij}\left[r_{ij}\right], \quad G_i = \tau_i\sum_{j=1}^{nm/\tilde{n}} K_{ij}\left[r_{ij}\right] \text{ and } \hat{t}_{ij}(\tau_i) \quad \forall j.$$

[14] The derivative of the regional FOC (1) w.r.t. ñ amounts to $E1 + E2$. The derivative of the local FOC (2) w.r.t. ñ amounts to $E3$.

[15] *HORIREG* amounts to the local SOC, *HORILOC* amounts to the regional SOC and *VERTI* amounts to the cross-partial derivative.

[16] Contrary to Konrad and Schjelderup (1999), all regions would be partitioned into coalitions, i.e., $\tilde{n}$ exogenous coalitions of $\frac{n}{\tilde{n}}$ regions.

[17] We know that $\frac{\partial \hat{t}_{ij}}{\partial \tau_i} \in ]-1;0[$ since $\left(\frac{\tilde{n}-1}{\tilde{n}} / \frac{nm-1}{nm}\right) < 1$, $2v' > \frac{nm-1}{nm}$, from Eq. (6) and $V' < v' = \alpha V'$ from Lemma 1 (see Appendix A).

[18] See for instance Fox et al. (2010).

Let us use the exponent $SR$ (resp. $SL$) to characterize the equilibrium when regional (resp. local) governments are leaders and the exponent $N$ to characterize the Nash equilibrium. The comparison between $\tau_i^{SR}$ and $\tau_i^N$ and the comparison between $t_{ij}^{SR}$ and $t_{ij}^N$ both crucially depend on the relative extent of marginal utilities. For $\frac{nm}{\tilde{n}}V' > v'$, regional tax rates chosen at the Stackelberg equilibrium are higher than those chosen at the Nash equilibrium, i.e., $\tau_i^{SR} > \tau_i^N$ $\forall i$. Because of the substitutability between local and regional tax rates, equilibrium tax rates chosen by local governments when they are followers are lower, i.e., $t_{ij}^{SR} < t_{ij}^N$ $\forall i, \forall j$. For $\frac{nm}{\tilde{n}}V' < v'$, results are opposite (see Appendix D for proof): when the regional public good valuation is low with respect to the local public good valuation, the regional leader sets a tax rate which is lower than the tax rate that would be chosen in a Nash game and thus produces less regional public good in order to maximize the welfare of the representative citizen, being aware that it will increase the local tax rate, and as a consequence the local public good provision. Regional governments thus do not always use their strategic advantage to set higher regional tax rates: the behavior of the regional leader will depend on whether an increase in the regional public good or an increase in the local public good generates more additional utility to citizens.

We then calculate the impact of the merger of regions on both regional and local tax rates by differentiating the system of first-order conditions w.r.t. $\tau_i$, $t_{ij}$ and $\tilde{n}$, taking into account $\hat{t}_{ij}(\tau_i)$. The results are summarized by the following proposition:

**Proposition 2.** *In a Stackelberg game where regions are leaders, the merger of regions always increases regional tax rates and decreases local tax rates.*

**Proof.** See Appendix D

Although some new effects capturing the impact of the merger on the reaction function emerge in the Stackelberg game when regions are leaders, in addition to the three existing effects $E1$, $E2$ and $E3$, we are able to show that the merger of regions leads to higher regional tax rates and lower local tax rates at the symmetric equilibrium. Results derived in a Nash game for both local and regional tax rates are therefore robust to this change in sequence of decisions. However, we are unable to sign the consolidated impact of a merger of regions without assuming more restrictive conditions. Let us now reverse the sequence of decisions and assume local leadership rather than regional leadership.

### 4.2. Local governments are Stackelberg leaders

In the same way, we solve the game by backward induction to derive the subgame perfect Nash equilibrium. We first calculate the best response function of each regional follower by differentiating the first-order conditions (Eq. (1)) by $t_{ij}$ and $\tau_i$ for all $i = 1, \ldots, \tilde{n}$ and $j = 1, \ldots \frac{nm}{\tilde{n}}$. At the symmetric equilibrium,[19] we obtain[20]:

$$\frac{\partial \hat{\tau}_i}{\partial t_{ij}} = -\frac{\left(v' + V' - \frac{\partial r_{ij}}{\partial t_{ij}}\right)}{\left(2V' \frac{nm}{\tilde{n}} - \frac{\partial r_{ij}}{\partial \tau_i}\right)} \in ]-1; 0[ \quad \forall i, \forall j.$$

Regional reaction functions are also negative, which confirms that local and regional tax rates are strategic substitutes.

Each local government $ij$ selects the tax rate $t_{ij}$ that maximizes the utility of its representative citizen, anticipating the predicted response of the region $i$, i.e.,

$$\underset{t_{ij}}{Max} \quad c_{ij} + v\left[g_{ij}\right] + V[G_i]$$

$$s.t. \quad c_{ij} = \Pi_{ij}\left[r_{ij}\right] + \rho\bar{k}, \quad g_{ij} = t_{ij}K_{ij}\left[r_{ij}\right],$$

$$G_i = \tau_i \sum_{j=1}^{nm/\tilde{n}} K_{ij}\left[r_{ij}\right] \text{ and } \hat{\tau}_i\left(t_{ij}\right) \quad \forall j.$$

As shown in Appendix E, the Stackelberg strategy is equivalent to the Nash strategy, i.e., local FOCs when cities are Stackelberg leaders boil down to local FOCs when cities play simultaneously with regions. Their first move does not give cities an advantage, and therefore, tax rates chosen at the equilibrium in the sequential game are identical to those chosen in the simultaneous game. This strong result is explained by a mix of two ingredients: the symmetry and the overlapping structure. Because of symmetry and provided that both local and regional governments are benevolent and utilitarian, the regional objective function is equivalent to the local objective function multiplied by the number $nm/\tilde{n}$ of cities inside the region. Therefore, the derivative of the regional objective function w.r.t. $\tau_i$ is identical to the derivative of the local objective function w.r.t. $\tau_i$ weighted by $\frac{nm}{\tilde{n}}$. This is the case only because the regional tax rate $\tau_i$ has the same impact for all cities inside the region since it is a top-tier policy.[21] The anticipation of the predicted response thus leads local leaders to not alter their best response, since it mimics the action of the regional follower. Cities are not better off in this case than they are in the simultaneous move case. As summarized by the following proposition, results are the same as in the Nash game:

**Proposition 3.** *Local leadership does not affect the outcome of the Nash game; that is, tax rates are identical* ($\tau_i^{SL} = \tau_i^N$, $t_{ij}^{SL} = t_{ij}^N$) *and the impact on taxation of the merger is the same* $\left(\frac{\partial t_{ij}^{SL}}{\partial \tilde{n}} = \frac{\partial t_{ij}^N}{\partial \tilde{n}} > 0, \frac{\partial \tau_i^{SL}}{\partial \tilde{n}} = \frac{\partial \tau_i^N}{\partial \tilde{n}} < 0, \frac{\partial T_{ij}^{SL}}{\partial \tilde{n}} = \frac{\partial T_{ij}^N}{\partial \tilde{n}} < 0\right)$.

**Proof.** See Appendix E

Results derived in a Nash game are therefore robust to this change in sequence of decisions.

## 5. Conclusion

In a two-tier tax competition model with several top-tier jurisdictions, which generates: i) horizontal tax externalities at both top and bottom tiers and ii) top-down and bottom-up vertical tax externalities, our paper analyzes the impact of a merger of top-tier jurisdictions on tax policies. Two top-tier (or regional) effects and one bottom-tier (or local) effect, both of which are horizontally and vertically transmitted, result from the merger. The two regional effects are found to overcome the local effect. Therefore, the merger of regions increases regional tax rates while decreasing local tax rates, with an

[19] Before simplification, the differentiated equation is $-K'_{ij}\left(\frac{\partial r_{ij}}{\partial \tau_i}d\tau_i + \frac{\partial r_{ij}}{\partial t_{ij}}dt_{ij}\right)\frac{\partial r_{ij}}{\partial \tau_i} + v'K'_{ij}\frac{\partial r_{ij}}{\partial \tau_i}dt_{ij} + V'\left(\sum_{j=1}^{nm/\tilde{n}} K'_{ij}\frac{\partial r_{ij}}{\partial \tau_i}d\tau_i + \sum_{k\neq j} K'_{ik}\frac{\partial \rho}{\partial t_{ij}}dt_{ij} + K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}}dt_{ij} + \sum_{j=1}^{nm/\tilde{n}} K'_{ij}\frac{\partial r_{ij}}{\partial \tau_i}d\tau_i\right) = 0.$

[20] We know that $\frac{\partial \hat{\tau}_i}{\partial t_{ij}} < 0$ because $\frac{nm}{\tilde{n}}V' > 1 > \frac{\partial r_{ij}}{\partial \tau_i}$ and $v' = \alpha V' > 1 > \frac{\partial r_{ij}}{\partial t_{ij}}$ from Lemma 2 (Appendix A), and that $\left|\frac{\partial \hat{\tau}_i}{\partial t_{ij}}\right| < 1$, which we proved in Appendix C.

[21] With regional leadership, this equivalence no longer holds because the region takes into account the impact of a given city's tax choice on the city itself and also on the other cities located in its territory. Therefore, the regional leader internalizes part of the local externalities when picking its tax rate first, whereas the city does not. Mathematically speaking, the derivative of the regional objective function w.r.t. $t_{ij}$ is not identical to the derivative of the local objective function w.r.t. $t_{ij}$.

overall positive impact on the consolidated tax rate. In case of a sequential move, with either local or regional leadership, the merger still exacerbates the race to the bottom at the local tier while reducing it at the regional tier. The equilibrium with local leadership has the special feature of being equivalent to the equilibrium with simultaneous moves due to the overlapping structure of taxation in our symmetric framework.

Several extensions can be considered to improve our understanding of the consequences of a merger of regions on tax competition in this two-tier framework, and thus to fuel the debate on territorial reorganization that has taken place in most OECD countries. It would be desirable to allow for more general functional forms of the utility function and the production function, without being obliged to introduce a new set of strong ad-hoc assumptions in compensation. The symmetry assumption could also be relaxed by considering that regions differ in terms of population or that only some regions merge. The determinants of the merger may also be endogenous, which would raise new issues concerning the stability of mergers of jurisdictions. Finally, future research could consider other types of territorial reforms such as the merger of jurisdictions that belong to two different tiers, for example, the merger of a region and some cities.

## Appendix A. Conditions of concavity and positivity

- Conditions of concavity

The concavity of the regional government's problem is ensured when the SOC $\left(2\frac{nm}{\tilde{n}}V'-\frac{\partial r_{ij}}{\partial \tau_i}\right)K'_{ij}\frac{\partial r_{ij}}{\partial \tau_i}\leq 0$ is satisfied. The concavity of the local government's problem is ensured when the SOC $\left(2v'-\frac{\partial r_{ij}}{\partial t_{ij}}\right)K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}}\leq 0$ is satisfied. Since $K'_{ij}<0, \frac{\partial r_{ij}}{\partial \tau_i}>0$, and $\frac{\partial r_{ij}}{\partial t_{ij}}>0$, the SOCs are satisfied under the two following assumptions:

$$2\frac{nm}{\tilde{n}}V'\geq\frac{\partial r_{ij}}{\partial \tau_i}, \tag{5}$$

$$2v'\geq\frac{\partial r_{ij}}{\partial t_{ij}}. \tag{6}$$

- Conditions of positivity

At the symmetric equilibrium, the amount of capital $K_{ij}$ invested in each city $ij$ is equal to the exogenous amount of capital $\overline{k}$ each representative citizen is initially endowed with. Using $K_{ij}=\overline{k}\ \forall i, \forall j$, we know that $\frac{\partial c_{ij}}{\partial \tau_i}=\frac{\partial c_{ij}}{\partial t_{ij}}=\Pi'_{ij}=-\overline{k}$ and $K'_{ij}=\frac{1}{F''[\overline{k}]}\forall i, \forall j$. The FOCs (1) and (2) thus reduce to:

$$\begin{cases} -\overline{k}+v't_{ij}K'_{ij}\dfrac{\partial r_{ij}}{\partial \tau_i}+V'\left(\dfrac{nm}{\tilde{n}}\overline{k}+\tau_i\dfrac{nm}{\tilde{n}}K'_{ij}\dfrac{\partial r_{ij}}{\partial \tau_i}\right)=0, \\ -\overline{k}+v'\left(\overline{k}+t_{ij}K'_{ij}\dfrac{\partial r_{ij}}{\partial t_{ij}}\right)+V'\tau_i\left(1+\dfrac{nm}{\tilde{n}}\dfrac{\partial \rho}{\partial t_{ij}}\right)K'_{ij}=0 \end{cases}$$

Solving this system of FOCs for all regions and cities simultaneously, we derive the tax rates chosen by regions and cities at the symmetric Nash equilibrium, denoted by $\tau_i^N$ and $t_{ij}^N$, $\forall i, \forall j$:

$$\tau_i^N=\frac{\left(\frac{nm}{\tilde{n}}V'-1\right)\left(\frac{nm-1}{nm}\Big/\frac{\tilde{n}-1}{\tilde{n}}\right)-(v'-1)}{-V'K'_{ij}\frac{nm-\tilde{n}}{\tilde{n}}}\overline{k}\quad and \tag{7}$$
$$t_{ij}^N=\frac{(v'-1)\frac{nm}{\tilde{n}}-\left(\frac{nm}{\tilde{n}}V'-1\right)}{-v'K'_{ij}\frac{nm-\tilde{n}}{\tilde{n}}}\overline{k}$$

As tax rates must be positive by assumption to ensure that public goods are provided, the following conditions of positivity are required[22] $\forall i, \forall j$:

$$\left(\frac{nm}{\tilde{n}}V'-1\right)\left(\frac{nm-1}{nm}\Big/\frac{\tilde{n}-1}{\tilde{n}}\right)\geq(v'-1) \tag{8}$$

$$(v'-1)\geq\left(\frac{nm}{\tilde{n}}V'-1\right)\frac{\tilde{n}}{nm}. \tag{9}$$

It should be noted that we checked that the set of parameters that satisfies conditions of concavity and conditions of positivity is not empty.

- Implications of the conditions of positivity

**Lemma 1.** $\alpha>1$

**Proof.** Replacing $v'$ by its value $\alpha V'$, condition (9) reduces to $(\alpha-1)\frac{nm}{\tilde{n}}V'\geq\frac{nm}{\tilde{n}}-1$. Because $\frac{nm}{\tilde{n}}-1>0$, we know that $\alpha$ can never be lower than 1. □

**Lemma 2.** $\frac{nm}{\tilde{n}}V'>1$ *and* $v'>1$

**Proof.** Combining the conditions of positivity (8) and (9), we know that $\left(\frac{nm}{\tilde{n}}V'-1\right)\left(\frac{nm-1}{nm}/\frac{\tilde{n}-1}{\tilde{n}}\right)\geq\left(\frac{nm}{\tilde{n}}V'-1\right)\frac{\tilde{n}}{nm}$, which boils down to the condition $\left(\frac{nm}{\tilde{n}}V'-1\right)\left(\frac{nm-\tilde{n}}{\tilde{n}-1}\right)\geq 0$. Because $nm>\tilde{n}$, we deduce that $\frac{nm}{\tilde{n}}V'>1$. Therefore, from condition (9), we can infer that $v'>1$. □

## Appendix B. Comparative statics

Differentiating the first-order conditions (1) and (2) with respect to $\tau_i$, $t_{ij}$ and $\tilde{n}$ yields the following system of equations in matrix form:

$$\begin{bmatrix} \left(2V'\dfrac{nm}{\tilde{n}}-\dfrac{\partial r_{ij}}{\partial \tau_i}\right)K'_{ij}\dfrac{\partial r_{ij}}{\partial \tau_i} & \begin{array}{l} -K'_{ij}\dfrac{\partial r_{ij}}{\partial t_{ij}}\dfrac{\partial r_{ij}}{\partial \tau_i}+v'K'_{ij}\dfrac{\partial r_{ij}}{\partial \tau_i} \\ +V'\left(\sum\limits_{k\neq j}K'_{ik}\dfrac{\partial \rho}{\partial t_{ij}}+K'_{ij}\dfrac{\partial r_{ij}}{\partial t_{ij}}\right)\end{array} \\ \begin{array}{l} -K'_{ij}\dfrac{\partial r_{ij}}{\partial \tau_i}\dfrac{\partial r_{ij}}{\partial t_{ij}}+v'K'_{ij}\dfrac{\partial r_{ij}}{\partial \tau_i} \\ +V'\left(\sum\limits_{k\neq j}K'_{ik}\dfrac{\partial \rho}{\partial t_{ij}}+K'_{ij}\dfrac{\partial r_{ij}}{\partial t_{ij}}\right)\end{array} & \left(2v'-\dfrac{\partial r_{ij}}{\partial t_{ij}}\right)K'_{ij}\dfrac{\partial r_{ij}}{\partial t_{ij}} \end{bmatrix}\begin{bmatrix}\dfrac{\partial \tau_i^N}{\partial \tilde{n}} \\ \dfrac{\partial t_{ij}^N}{\partial \tilde{n}}\end{bmatrix}$$
$$=\begin{bmatrix} -v't_{ij}K'_{ij}\dfrac{\partial\left(\frac{\partial r_{ij}}{\partial \tau_i}\right)}{\partial \tilde{n}}-V'\dfrac{\partial\left(\left(1+\varepsilon_{\tau_i}\right)\sum_{j=1}^{nm/\tilde{n}}K_{ij}\right)}{\partial \tilde{n}} \\ -V'\tau_i\dfrac{\partial\left(\sum_{k\neq j}K'_{ik}\frac{\partial \rho}{\partial t_{ij}}+K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}}\right)}{\partial \tilde{n}}\end{bmatrix}.$$

Using Cramer's rule gives:

$$\frac{\partial \tau_i^N}{\partial \tilde{n}}=\frac{B^N}{A^N}\quad\text{and}\quad\frac{\partial t_{ij}^N}{\partial \tilde{n}}=\frac{C^N}{A^N},$$

[22] The need for conditions of positivity is explained by the linearity of the utility derived from local and regional public goods.

where[23]:

$$A^N = K_{ij}'^2 \left( \begin{array}{l} \left(2V'\frac{nm}{\tilde{n}} - \frac{\partial r_{ij}}{\partial \tau_i}\right)\frac{\partial r_{ij}}{\partial \tau_i}\left(2v' - \frac{\partial r_{ij}}{\partial t_{ij}}\right)\frac{\partial r_{ij}}{\partial t_{ij}} \\ -\left(-\frac{\partial r_{ij}}{\partial \tau_i}\frac{\partial r_{ij}}{\partial t_{ij}} + v'\frac{\partial r_{ij}}{\partial \tau_i} + V'\left(1 + \frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)\right)^2 \end{array} \right),$$

$$B^N = K_{ij}'^2 \left( \begin{array}{l} -\frac{\partial\left(\frac{\partial r_{ij}}{\partial \tau_i}\right)}{\partial \tilde{n}}\left(v't_{ij} + V'\frac{nm}{\tilde{n}}\tau_i\right)\left(2v' - \frac{\partial r_{ij}}{\partial t_{ij}}\right)\frac{\partial r_{ij}}{\partial t_{ij}} \\ +\frac{\partial\left(\frac{nm}{\tilde{n}}\right)}{\partial \tilde{n}}\frac{V'}{\left(-K_{ij}'\right)}\left(K_{ij} + \tau_i K_{ij}'\frac{\partial r_{ij}}{\partial \tau_i}\right)\left(2v' - \frac{\partial r_{ij}}{\partial t_{ij}}\right)\frac{\partial r_{ij}}{\partial t_{ij}} \\ +\frac{\partial\left(1 + \frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)}{\partial \tilde{n}}V'\tau_i\left(-\frac{\partial r_{ij}}{\partial t_{ij}}\frac{\partial r_{ij}}{\partial \tau_i} + v'\frac{\partial r_{ij}}{\partial \tau_i} + V'\left(1 + \frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)\right) \end{array} \right),$$

$$C^N = K_{ij}'^2 \left( \begin{array}{l} \frac{\partial\left(\frac{\partial r_{ij}}{\partial \tau_i}\right)}{\partial \tilde{n}}\left(v't_{ij} + V'\frac{nm}{\tilde{n}}\tau_i\right)\left(-\frac{\partial r_{ij}}{\partial \tau_i}\frac{\partial r_{ij}}{\partial t_{ij}} + v'\frac{\partial r_{ij}}{\partial \tau_i} + V'\left(1 + \frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)\right) \\ +\frac{\partial\left(\frac{nm}{\tilde{n}}\right)}{\partial \tilde{n}}\frac{V'}{K_{ij}'}\left(K_{ij} + \tau_i K_{ij}'\frac{\partial r_{ij}}{\partial \tau_i}\right)\left(-\frac{\partial r_{ij}}{\partial \tau_i}\frac{\partial r_{ij}}{\partial t_{ij}} + v'\frac{\partial r_{ij}}{\partial \tau_i} + V'\left(1 + \frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)\right) \\ -\frac{\partial\left(1 + \frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)}{\partial \tilde{n}}V'\tau_i\left(2V'\frac{nm}{\tilde{n}} - \frac{\partial r_{ij}}{\partial \tau_i}\right)\frac{\partial r_{ij}}{\partial \tau_i} \end{array} \right).$$

The expressions $B^N$ and $C^N$ can be rewritten as follows:

$$B^N = (E1 + E2) * HORIREG - E3 * VERTI,$$
$$C^N = -(E1 + E2) * VERTI + E3 * HORILOC,$$

with[24]

$$E1 = -\left(v't_{ij} + V'\frac{nm}{\tilde{n}}\tau_i\right)K_{ij}'\frac{\partial\left(\partial r_{ij}/\partial \tau_i\right)}{\partial \tilde{n}} > 0,$$

$$E2 = -V'\frac{\partial(nm/\tilde{n})}{\partial \tilde{n}}\left(K_{ij} + \tau_i K_{ij}'\frac{\partial r_{ij}}{\partial \tau_i}\right) > 0,$$

$$E3 = -V'\tau_i K_{ij}'\frac{\partial\left(1 + \frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)}{\partial \tilde{n}} > 0,$$

$$HORIREG = \left(2v' - \frac{\partial r_{ij}}{\partial t_{ij}}\right)K_{ij}'\frac{\partial r_{ij}}{\partial t_{ij}} < 0,$$

$$HORILOC = \left(2V'\frac{nm}{\tilde{n}} - \frac{\partial r_{ij}}{\partial \tau_i}\right)K_{ij}'\frac{\partial r_{ij}}{\partial \tau_i} < 0,$$

$$VERTI = \left(-\frac{\partial r_{ij}}{\partial t_{ij}}\frac{\partial r_{ij}}{\partial \tau_i} + v'\frac{\partial r_{ij}}{\partial \tau_i} + V'\left(1 + \frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)\right)K_{ij}' < 0.$$

We show that the sign of $A^N$ is always positive:

**Lemma 3.** $A^N > 0$

**Proof.** We proceed by contradiction. Assume that $A^N < 0$. Substituting the values for $\frac{\partial r_{ij}}{\partial \tau_i}, \frac{\partial r_{ij}}{\partial t_{ij}}, \frac{\partial \rho}{\partial t_{ij}}$ and using $v' = \alpha V'$, we obtain that $A^N < 0$ iif[25]:

$$\tilde{n} > 1 + 2(nm-1)\frac{\left(2\alpha V' - \frac{nm-1}{nm}\right)}{(\alpha+1)^2 V' - 2\frac{nm-1}{nm}}. \tag{11}$$

Condition (8) requires[26]:

$$\tilde{n} < \frac{V'(nm-1) + (\alpha V' - 1)}{\left(\alpha V' - \frac{1}{nm}\right)}, \tag{12}$$

to make sure that the regional tax rate is strictly positive. We can show that conditions (11) and (12) are mutually incompatible. Indeed, $1 + 2(nm-1)\frac{\left(2\alpha V' - \frac{nm-1}{nm}\right)}{\left((\alpha+1)^2 V' - 2\frac{nm-1}{nm}\right)} < \frac{V'(nm-1) + (\alpha V' - 1)}{\left(\alpha V' - \frac{1}{nm}\right)}$ iif $(((3\alpha + 1)V' - 2)nm + (\alpha + 1))(\alpha - 1)(nm - 1)V' < 0$, which is impossible from Lemmas 1 and 2 (stated in Appendix A). □

It should be noted that the solution is a steady state of our linear system of differential equations, as every real eigenvalue of $A^N > 0$ is negative.

## Appendix C. Proof of Proposition 1

Proofs refer to conditions and lemmas stated in Appendices A and B.

- Proof that $\frac{\partial \tau_i^N}{\partial \tilde{n}} < 0$
  Let $B^{N\prime} = K_{ij}'^2\left(-\frac{\partial\left(\partial r_{ij}/\partial \tau_i\right)}{\partial \tilde{n}}\left(v't_{ij} + V'\frac{nm}{\tilde{n}}\tau_i\right)\left(2v' - \frac{\partial r_{ij}}{\partial t_{ij}}\right)\frac{\partial r_{ij}}{\partial t_{ij}} + \frac{\partial\left(1 + \frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)}{\partial \tilde{n}} * V'\tau_i\left(-\frac{\partial r_{ij}}{\partial \tau_i}\frac{\partial r_{ij}}{\partial t_{ij}} + v'\frac{\partial r_{ij}}{\partial \tau_i} + V'\left(1 + \frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)\right)\right)$ such that $B^N - B^{N\prime} = K_{ij}'^2 * \frac{\partial(nm/\tilde{n})}{\partial \tilde{n}}\frac{V'}{\left(-K_{ij}'\right)}\left(K_{ij} + \tau_i K_{ij}'\frac{\partial r_{ij}}{\partial \tau_i}\right)\left(2v' - \frac{\partial r_{ij}}{\partial t_{ij}}\right)\frac{\partial r_{ij}}{\partial t_{ij}}$, which is always $<0$ from Eq. (6) and given $\varepsilon_{\tau_i} \in ]-1, 0[$. Demonstrating that $B^{N\prime}$ is negative ensures that $B^N$ is negative too. After replacing $\frac{\partial r_{ij}}{\partial \tau_i}, \frac{\partial r_{ij}}{\partial t_{ij}}$, and $\frac{\partial \rho}{\partial t_{ij}}$ by their values and substituting $v'$ by $\alpha V'$, the expression of $B^{N\prime}$ boils down to $K_{ij}'^2\frac{\partial\left(\frac{\tilde{n}-1}{\tilde{n}}\right)}{\partial \tilde{n}}V'\left(-\left(\alpha t_{ij} + \frac{nm}{\tilde{n}}\tau_i\right)\left(2\alpha V' - \frac{nm-1}{nm}\right)\frac{nm-1}{nm} + \tau_i\left(-\frac{nm-1}{nm} + (\alpha+1)V'\right)\frac{\tilde{n}-1}{\tilde{n}}\right)$. Because $K_{ij}'^2\frac{\partial\left(\frac{\tilde{n}-1}{\tilde{n}}\right)}{\partial \tilde{n}}V' > 0$, and $\left(2\alpha V' - \frac{nm-1}{nm}\right) > \left(-\frac{nm-1}{nm} + (\alpha+1)V'\right)$ from Lemma 1, a sufficient condition to ensure that $B^{N\prime}$ is negative is that $-\left(\alpha t_{ij} + \frac{nm}{\tilde{n}}\tau_i\right)\frac{nm-1}{nm} + \tau_i\frac{\tilde{n}-1}{\tilde{n}} < 0$ or, equally, $\alpha t_{ij}\frac{nm-1}{nm} + \tau_i\frac{nm-\tilde{n}}{\tilde{n}} > 0$, which is always true. Therefore $B^{N\prime}$ and consequently $B^N$ are negative. Because $A^N > 0$ from Lemma 3, $\frac{\partial \tau_i^N}{\partial \tilde{n}} = \frac{B^N}{A^N} < 0$.
  □
- Proof that $\frac{\partial t_{ij}^N}{\partial \tilde{n}} > 0$
  After replacing $\frac{\partial r_{ij}}{\partial \tau_i}, \frac{\partial r_{ij}}{\partial t_{ij}}$ and $\frac{\partial \rho}{\partial t_{ij}}$ by their values and substituting $v'$ by $\alpha V'$, the expression of $C^N$ simplifies as follows: $C^N = K_{ij}'^2 V'\frac{\tilde{n}-1}{\tilde{n}}\frac{1}{\tilde{n}^2}\left(\frac{1}{\tilde{n}}\tau_i\left(1 + 2nm(\alpha V' - 1)\right) + \left(\alpha t_{ij} + \frac{nm}{-K'_{ij}}\left(K_{ij} + \tau_i K_{ij}'\right)\right)\left(-\frac{nm-1}{nm} + (\alpha+1)V'\right) + \tau_i\right)$ which implies that $C^N > 0$ iif $\tilde{n}\left(\left(\alpha t_{ij} + \frac{nm}{-K'_{ij}}\left(K_{ij} + \tau_i K_{ij}'\right)\right)\left(-\frac{nm-1}{nm} + (\alpha+1)V'\right) + \tau_i\right) > -\tau_i\left(1 + 2nm\,(\alpha V' - 1)\right)$. We know from Lemma 2 that the right-hand side is negative. Given $\varepsilon_{\tau_i} \in ]-1, 0[$ and $\left(-\frac{nm-1}{nm} + (\alpha+1)V'\right) > 0$ from Lemma 2, the condition is always true because $\tilde{n} > 0$. Therefore $C^N$ is positive. Because $A^N > 0$ from Lemma 3, $\frac{\partial t_{ij}^N}{\partial \tilde{n}} = \frac{C^N}{A^N} > 0$. □

[23] Invoking symmetry, i.e. $K_{ij} = \overline{k}$ and $K_{ij}' = 1/F''\left[\overline{k}\right]\forall i, \forall j$, we simplify the following expressions: $\partial\left((1 + \varepsilon_{\tau_i})\sum_{j=1}^{nm/\tilde{n}}K_{ij}\right)\Big/\partial \tilde{n} = \left(\partial\frac{nm}{\tilde{n}}/\partial \tilde{n}\right)\left(K_{ij} + \tau_i K_{ij}'\frac{\partial r_{ij}}{\partial \tau_i}\right) + \sum_{j=1}^{nm/\tilde{n}}\tau_i K_{ij}'\left(\partial\frac{\partial r_{ij}}{\partial \tau_i}/\partial \tilde{n}\right)$ and $\left(\sum_{k \neq j}K_{ik}'\frac{\partial \rho}{\partial t_{ij}} + K_{ij}'\frac{\partial r_{ij}}{\partial t_{ij}}\right) = \left(1 + \frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)$.

[24] We know that $HORILOC < 0$ from Eq. (5), $HORIREG < 0$ from Eq. (6) and $VERTI = \left(-\frac{\partial r_{ij}}{\partial t_{ij}}\frac{\partial r_{ij}}{\partial \tau_i} + v'\frac{\partial r_{ij}}{\partial \tau_i} + V'\left(1 + \frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)\right)K_{ij}' = \left(-\frac{\partial r_{ij}}{\partial t_{ij}} + v' + V'\right)\left(\frac{\tilde{n}-1}{\tilde{n}}\right)K_{ij}' < 0$ from Lemma 2.

[25] $(\alpha+1)^2 V' - 2\frac{nm-1}{nm} > 0$ because it is the sum of two positive terms, i.e., $(\alpha^2 + 1)V' - \frac{nm-1}{nm}$, which is positive from Eq. (6) and Lemma 1, and $2\alpha V' - \frac{nm-1}{nm}$, which is positive from Eq. (6).

[26] $\left(\alpha V' - \frac{1}{nm}\right) > 0$ from Eq. (6) and provided that $nm > 3$, which is always true.

- Proof that $\frac{\partial T_{ij}^N}{\partial \tilde{n}}<0$

  Summing $\frac{\partial \tau_i^N}{\partial \tilde{n}}$ and $\frac{\partial t_{ij}^N}{\partial \tilde{n}}$, and simplifying, we get:

$$\frac{\partial T_{ij}^N}{\partial \tilde{n}}=\frac{1}{A^N\tilde{n}^2}K_{ij}^{\prime 2}\left(\begin{array}{l}\left(v' t_{ij}+V'\frac{nm}{\tilde{n}}\tau_i+nm\frac{V'}{-K'_{ij}}\left(K_{ij}+\tau_i K'_{ij}\frac{\partial r_{ij}}{\partial \tau_i}\right)\right)\\ \times\left(\left(-\frac{\partial r_{ij}}{\partial t_{ij}}+v'+V'\right)\frac{\tilde{n}-1}{\tilde{n}}-\left(2v'-\frac{\partial r_{ij}}{\partial t_{ij}}\right)\frac{\partial r_{ij}}{\partial t_{ij}}\right)\\ +V'\tau_i\frac{\tilde{n}-1}{\tilde{n}}\left(\left(-\frac{\partial r_{ij}}{\partial t_{ij}}+v'+V'\right)-\left(2V'\frac{nm}{\tilde{n}}-\frac{\partial r_{ij}}{\partial \tau_i}\right)\right)\end{array}\right).$$

The first expression into the bracket, $\left(v' t_{ij}+V'\frac{nm}{\tilde{n}}\tau_i+nm\frac{V'}{-K'_{ij}}\left(K_{ij}+\tau_i K'_{ij}\frac{\partial r_{ij}}{\partial \tau_i}\right)\right)\left(\left(-\frac{\partial r_{ij}}{\partial t_{ij}}+v'+V'\right)\frac{\tilde{n}-1}{\tilde{n}}-\left(2v'-\frac{\partial r_{ij}}{\partial t_{ij}}\right)\frac{\partial r_{ij}}{\partial t_{ij}}\right)$, is negative because i) $\left(v' t_{ij}+V'\frac{nm}{\tilde{n}}\tau_i+nm\frac{V'}{(-K'_{ij})}\left(K_{ij}+\tau_i K'_{ij}\frac{\partial r_{ij}}{\partial \tau_i}\right)\right)$ is always positive and ii) $\left(-\frac{\partial r_{ij}}{\partial t_{ij}}+v'+V'\right)\frac{\tilde{n}-1}{\tilde{n}}-\left(2v'-\frac{\partial r_{ij}}{\partial t_{ij}}\right)\frac{\partial r_{ij}}{\partial t_{ij}}$ is negative since $\alpha>1$ from Lemma 1 and $\frac{nm-1}{nm}>\frac{\tilde{n}-1}{\tilde{n}}$.

The second expression into the bracket, $V'\tau_i\frac{\tilde{n}-1}{\tilde{n}}\left(\left(-\frac{\partial r_{ij}}{\partial t_{ij}}+v'+V'\right)-\left(2V'\frac{nm}{\tilde{n}}-\frac{\partial r_{ij}}{\partial \tau_i}\right)\right)$, is also negative. Indeed, we can show that the term $\left(-\frac{\partial r_{ij}}{\partial t_{ij}}+v'+V'\right)-\left(2V'\frac{nm}{\tilde{n}}-\frac{\partial r_{ij}}{\partial \tau_i}\right)$ is negative iif $v'-1\leq-\frac{\partial r_{ij}}{\partial \tau_i}+2V'\frac{nm}{\tilde{n}}+\frac{\partial r_{ij}}{\partial t_{ij}}-V'-1$ (substracting $-1$ on each side). From Eq. (8), we know that $v'-1\leq\left(\frac{nm}{\tilde{n}}V'-1\right)\left(\frac{\partial r_{ij}}{\partial t_{ij}}/\frac{\partial r_{ij}}{\partial \tau_i}\right)$. A sufficient condition for $\left(-\frac{\partial r_{ij}}{\partial t_{ij}}+v'+V'\right)-\left(2V'\frac{nm}{\tilde{n}}-\frac{\partial r_{ij}}{\partial \tau_i}\right)$ to be $\leq 0$ is that $-\frac{\partial r_{ij}}{\partial \tau_i}+2V'\frac{nm}{\tilde{n}}+\frac{\partial r_{ij}}{\partial t_{ij}}-V'-1\geq\left(\frac{nm}{\tilde{n}}V'-1\right)\left(\frac{\partial r_{ij}}{\partial t_{ij}}/\frac{\partial r_{ij}}{\partial \tau_i}\right)$ or equally $0\geq\frac{-1}{\frac{\partial r_{ij}}{\partial \tau_i}}\left(V'\left(\frac{nm}{\tilde{n}}-1\right)\frac{\tilde{n}-2}{\tilde{n}}+\left(\frac{\partial r_{ij}}{\partial t_{ij}}-\frac{\partial r_{ij}}{\partial \tau_i}\right)*\left(1+\frac{\partial r_{ij}}{\partial \tau_i}\right)\right)$ after factorization and simplification, which is always true since $\frac{nm}{\tilde{n}}>1, \tilde{n}\geq 2$ and $\frac{\partial r_{ij}}{\partial t_{ij}}>\frac{\partial r_{ij}}{\partial \tau_i}$. Because $A^N>0$ from Lemma 3, $\frac{\partial T_{ij}^N}{\partial \tilde{n}}$ is the sum of negative expressions. Therefore, $\frac{\partial T_{ij}}{\partial \tilde{n}}<0$. □

## Appendix D. Regional leadership

- Derivation of the equilibrium tax rates

  Solving the regional government's program when she/he acts as a Stackelberg leader, we obtain the following regional FOC:

$$\sum_{j=1}^{nm/\tilde{n}}\left(\begin{array}{l}\Pi'_{ij}\left(\frac{\partial r_{ij}}{\partial \tau_i}+\frac{\partial r_{ij}}{\partial t_{ij}}\frac{\partial \hat{t}_{ij}}{\partial \tau_i}+\sum_{k\neq j}\frac{\partial \rho}{\partial t_{ik}}\frac{\partial t_{ik}^F}{\partial \tau_i}\right)+\left(\frac{\partial \rho}{\partial \tau_i}+\frac{\partial \rho}{\partial t_{ij}}\frac{\partial \hat{t}_{ij}}{\partial \tau_i}+\sum_{k\neq j}\frac{\partial \rho}{\partial t_{ik}}\frac{\partial t_{ik}^F}{\partial \tau_i}\right)\bar{k}\\ +v'\left(\frac{\partial \hat{t}_{ij}}{\partial \tau_i}K_{ij}+t_{ij}K'_{ij}\left(\frac{\partial r_{ij}}{\partial \tau_i}+\frac{\partial r_{ij}}{\partial t_{ij}}\frac{\partial \hat{t}_{ij}}{\partial \tau_i}+\sum_{k\neq j}\frac{\partial \rho}{\partial t_{ik}}\frac{\partial t_{ik}^F}{\partial \tau_i}\right)\right)\\ +V'\left(\sum_{j=1}^{nm/\tilde{n}}K_{ij}+\tau_i\sum_{j=1}^{nm/\tilde{n}}K'_{ij}\left(\frac{\partial r_{ij}}{\partial \tau_i}+\frac{\partial r_{ij}}{\partial t_{ij}}\frac{\partial \hat{t}_{ij}}{\partial \tau_i}+\sum_{k\neq j}\frac{\partial \rho}{\partial t_{ik}}\frac{\partial t_{ik}^F}{\partial \tau_i}\right)\right)\end{array}\right)=0. \quad (13)$$

At the symmetric equilibrium, using $K_{ij}=\bar{k}\ \forall i, \forall_j$, FOCs (13) and (2) reduce to:

$$\begin{cases}\left(v'\frac{\partial \hat{t}_{ij}}{\partial \tau_i}+V'\frac{nm}{\tilde{n}}-\left(1+\frac{\partial \hat{t}_{ij}}{\partial \tau_i}\right)\right)\bar{k}+v' t_{ij}K'_{ij}\left(\frac{\partial r_{ij}}{\partial \tau_i}+\left(1+\frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)\frac{\partial \hat{t}_{ij}}{\partial \tau_i}\right)\\ \quad +V'\tau_i\frac{nm}{\tilde{n}}K'_{ij}\left(\frac{\partial r_{ij}}{\partial \tau_i}+\left(1+\frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)\frac{\partial \hat{t}_{ij}}{\partial \tau_i}\right)=0\\ (v'-1)\bar{k}+v' t_{ij}K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}}+V'\tau_i\left(\frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ij}}+1\right)K'_{ij}=0.\end{cases}$$

Solving this system of FOCs for all regions and cities, we derive the tax rates chosen by regions and cities at the symmetric equilibrium when regional governments are Stackelberg leaders[27]:

$$\tau_i^{SR}=\frac{\left(\frac{nm}{\tilde{n}}V'-1\right)\left(\frac{nm-1}{nm}\Big/\frac{\tilde{n}-1}{\tilde{n}}\right)-(v'-1)\left(1-\left(\left(\frac{nm-1}{nm}\Big/\frac{\tilde{n}-1}{\tilde{n}}\right)-1\right)\frac{\partial \hat{t}_{ij}}{\partial \tau_i}\right)}{-V'K'_{ij}\frac{nm-\tilde{n}}{\tilde{n}}\left(1+\frac{\partial \hat{t}_{ij}}{\partial \tau_i}\right)}\bar{k}$$

$$t_{ij}^{SR}=\frac{(v'-1)\left(\frac{nm}{\tilde{n}}\left(1+\frac{\partial \hat{t}_{ij}}{\partial \tau_i}\right)-\frac{\partial \hat{t}_{ij}}{\partial \tau_i}\right)-\left(\frac{nm}{\tilde{n}}V'-1\right)}{-v'K'_{ij}\frac{nm-\tilde{n}}{\tilde{n}}\left(1+\frac{\partial \hat{t}_{ij}}{\partial \tau_i}\right)}\bar{k}.$$

For $\frac{nm}{\tilde{n}}V'>v'$, we show that $\tau_i^{SR}>\tau_i^N$ and $t_{ij}^{SR}<t_{ij}^N$. In contrast, for $\frac{nm}{\tilde{n}}V'<v'$, we show that $\tau_i^{SR}<\tau_i^N$ and $t_{ij}^{SR}>t_{ij}^N$. It should be noted that we checked that conditions of positivity and conditions of concavity stated in the Nash game are compatible with those in the Stackelberg game.

- Comparative statics

  Differentiating FOCs (13) and (2) w.r.t. $\tau_i$, $t_{ij}$ and ñ in this Stackelberg game yields the following system of equations in matrix form:

$$\begin{bmatrix}\begin{array}{l}\left(2V'\frac{nm}{\tilde{n}}+2v'\frac{\partial \hat{t}_{ij}}{\partial \tau_i}-\left(\frac{\partial r_{ij}}{\partial \tau_i}+\left(1+\frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ik}}\right)\frac{\partial \hat{t}_{ij}}{\partial \tau_i}\right)\right)\\ *K'_{ij}\left(\frac{\partial r_{ij}}{\partial \tau_i}+\left(1+\frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)\frac{\partial \hat{t}_{ij}}{\partial \tau_i}\right)\end{array} & \begin{array}{l}-K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}}\left(\frac{\partial r_{ij}}{\partial \tau_i}+\left(1+\frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ik}}\right)\frac{\partial \hat{t}_{ij}}{\partial \tau_i}\right)\\ +v'K'_{ij}\left(\frac{\partial \hat{t}_{ij}}{\partial \tau_i}\frac{\partial r_{ij}}{\partial t_{ij}}+\left(\frac{\partial r_{ij}}{\partial \tau_i}+\left(1+\frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)\frac{\partial \hat{t}_{ij}}{\partial \tau_i}\right)\right)\\ +V'K'_{ij}\left(1+\frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)\end{array}\\ \begin{array}{l}-K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}}\frac{\partial r_{ij}}{\partial \tau_i}+v'K'_{ij}\frac{\partial r_{ij}}{\partial \tau_i}\\ +V'K'_{ij}\left(1+\frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)\end{array} & \left(2v'-\frac{\partial r_{ij}}{\partial t_{ij}}\right)K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}}\end{bmatrix}\times\begin{bmatrix}\frac{\partial \tau_i^{SR}}{\partial \tilde{n}}\\ \frac{\partial t_{ij}^{SR}}{\partial \tilde{n}}\end{bmatrix}$$

$$=\begin{bmatrix}-\left(\begin{array}{l}(v'-1)\frac{\partial\frac{\partial \hat{t}_{ij}}{\partial \tau_i}}{\partial \tilde{n}}\bar{k}+v' t_{ij}K'_{ij}\left(\frac{\partial\frac{\partial r_{ij}}{\partial \tau_i}}{\partial \tilde{n}}+\frac{\partial\left(\frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)}{\partial \tilde{n}}\frac{\partial \hat{t}_{ij}}{\partial \tau_i}+\left(1+\frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)\frac{\partial\frac{\partial \hat{t}_{ij}}{\partial \tau_i}}{\partial \tilde{n}}\right)\\ +V'\frac{\partial\frac{nm}{\tilde{n}}}{\partial \tilde{n}}\left(K_{ij}+\tau_i K'_{ij}\left(\frac{\partial r_{ij}}{\partial \tau_i}+\left(1+\frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)\frac{\partial \hat{t}_{ij}}{\partial \tau_i}\right)\right)\\ +V'\frac{nm}{\tilde{n}}\tau_i K'_{ij}\left(\frac{\partial\frac{\partial r_{ij}}{\partial \tau_i}}{\partial \tilde{n}}+\frac{\partial\left(\frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)}{\partial \tilde{n}}\frac{\partial \hat{t}_{ij}}{\partial \tau_i}+\left(1+\frac{nm}{\tilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)\frac{\partial\frac{\partial \hat{t}_{ij}}{\partial \tau_i}}{\partial \tilde{n}}\right)-V'\tau_i K'_{ij}\frac{\partial\frac{nm}{\tilde{n}}}{\partial \tilde{n}}\frac{\partial \rho}{\partial t_{ij}}\end{array}\right)\end{bmatrix}$$

Using Cramer's rule gives:

$$\frac{\partial \tau_i^{SR}}{\partial \tilde{n}}=\frac{B^{SR}}{A^{SR}}\text{ and }\frac{\partial t_{ij}^{SR}}{\partial \tilde{n}}=\frac{C^{SR}}{A^{SR}},$$

where after simplification due to symmetry, and using $B^N$ and $C^N$ defined in Appendix B, we get:

$$A^{SR}=K_{ij}^{\prime 2}\left(\begin{array}{l}\left(2V'\frac{nm}{\tilde{n}}+2v'\frac{\partial \hat{t}_{ij}}{\partial \tau_i}-\frac{\tilde{n}-1}{\tilde{n}}\left(1+\frac{\partial \hat{t}_{ij}}{\partial \tau_i}\right)\right)\frac{\tilde{n}-1}{\tilde{n}}\left(1+\frac{\partial \hat{t}_{ij}}{\partial \tau_i}\right)\left(2v'-\frac{\partial r_{ij}}{\partial t_{ij}}\right)\frac{\partial r_{ij}}{\partial t_{ij}}\\ -\frac{\tilde{n}-1}{\tilde{n}}\left(v'+V'-\frac{\partial r_{ij}}{\partial t_{ij}}\right)\\ \times\left(-\frac{\partial r_{ij}}{\partial t_{ij}}\frac{\tilde{n}-1}{\tilde{n}}\left(1+\frac{\partial \hat{t}_{ij}}{\partial \tau_i}\right)+v'\left(\left(\frac{\partial \hat{t}_{ij}}{\partial \tau_i}\frac{\partial r_{ij}}{\partial t_{ij}}+\frac{\tilde{n}-1}{\tilde{n}}\left(1+\frac{\partial \hat{t}_{ij}}{\partial \tau_i}\right)\right)+V'\frac{\tilde{n}-1}{\tilde{n}}\right)\right)\end{array}\right)$$

$$B^{SR}=B^N\left(1+\frac{\partial \hat{t}_{ij}}{\partial \tau_i}\right)+K_{ij}^{\prime 2}\left(-D+\frac{\partial\frac{nm}{\tilde{n}}}{\partial \tilde{n}}\frac{\partial \rho}{\partial t_{ij}}V'\tau_i\left(v'\frac{\partial r_{ij}}{\partial t_{ij}}-V'\frac{\tilde{n}-1}{\tilde{n}}\right)\frac{\partial \hat{t}_{ij}}{\partial \tau_i}\right)$$

$$C^{SR}=C^N\left(1+\frac{\partial \hat{t}_{ij}}{\partial \tau_i}\right)+K_{ij}^{\prime 2}\left(-D\frac{\partial \hat{t}_{ij}}{\partial \tau_i}-\frac{\partial\frac{nm}{\tilde{n}}}{\partial \tilde{n}}\frac{\partial \rho}{\partial t_{ij}}V'\tau_i\left(2v'-\frac{\tilde{n}-1}{\tilde{n}}\right)\frac{\partial \hat{t}_{ij}}{\partial \tau_i}\frac{\tilde{n}-1}{\tilde{n}}\left(1+\frac{\partial \hat{t}_{ij}}{\partial \tau_i}\right)\right)$$

where $D\equiv\left(\frac{nm}{\tilde{n}}\frac{V'}{-K'_{ij}}\left(\bar{k}+\tau_i K'_{ij}\frac{\tilde{n}-1}{\tilde{n}}\right)+v'\frac{1}{-K'_{ij}}\left(\bar{k}+t_{ij}K'_{ij}\frac{\tilde{n}-1}{\tilde{n}}\right)+\left((\tilde{n}-2)\frac{nm}{\tilde{n}}V'-1\right)\frac{\bar{k}}{-K'_{ij}}\right)\frac{1}{\tilde{n}^2}\left(v'+V'-\frac{\partial r_{ij}}{\partial t_{ij}}\right)$.

[27] For $\frac{\partial \hat{t}_{ij}}{\partial \tau_i}=0$, we find the same results as in the Nash game.

- Signs of $\frac{\partial \tau_i^{SR}}{\partial \tilde{n}}$ and $\frac{\partial t_{ij}^{SR}}{\partial \tilde{n}}$

  Using the condition that ensures the positivity of the regional tax rate, we show that $A^{SR} > 0$. The sign of $\frac{\partial \tau_i^{SR}}{\partial \tilde{n}}$ (resp. $\frac{\partial t_{ij}^{SR}}{\partial \tilde{n}}$) is thus given by the sign of $B^{SR}$ (resp. $C^{SR}$). Provided that $\tilde{n} > 2$, the term $D$ is always positive because we postulated that both $\varepsilon_{t_{ij}}$ and $\varepsilon_{\tau_i}$ belong to the interval$]-1, 0[$. Since $B^N < 0$ and $C^N > 0$ from Appendix C, $\left(1+\frac{\partial \hat{t}_{ij}}{\partial \tau_i}\right) > 0, D > 0, \frac{\partial \frac{nm}{\tilde{n}}}{\partial \tilde{n}} \frac{\partial \rho}{\partial t_{ij}} V' \tau_i \left(v' \frac{\partial r_{ij}}{\partial t_{ij}} - V' \frac{\tilde{n}-1}{\tilde{n}}\right) \frac{\partial \hat{t}_{ij}}{\partial \tau_i} < 0$ and $-\frac{\partial \frac{nm}{\tilde{n}}}{\partial \tilde{n}} \frac{\partial \rho}{\partial t_{ij}} V' \tau_i \left(2v' - \frac{\tilde{n}-1}{\tilde{n}}\right) \frac{\partial \hat{t}_{ij}}{\partial \tau_i} \frac{\tilde{n}-1}{\tilde{n}} \left(1+\frac{\partial \hat{t}_{ij}}{\partial \tau_i}\right) > 0$, we prove that $\frac{\partial \tau_i^{SR}}{\partial \tilde{n}} < 0$ and $\frac{\partial t_{ij}^{SR}}{\partial \tilde{n}} > 0$. □

## Appendix E. Local leadership

Solving the local government's program when she/he acts as a Stackelberg leader, we obtain the following FOC:

$$\begin{aligned} &\Pi_{ij}' \left( \frac{\partial r_{ij}}{\partial \tau_i} \frac{\partial \hat{\tau}_i}{\partial t_{ij}} + \frac{\partial r_{ij}}{\partial t_{ij}} \right) + \left( \frac{\partial \rho}{\partial \tau_i} \frac{\partial \hat{\tau}_i}{\partial t_{ij}} + \frac{\partial \rho}{\partial t_{ij}} \right) \overline{k} \\ &+ v' \left( K_{ij} + t_{ij} K_{ij}' \left( \frac{\partial r_{ij}}{\partial \tau_i} \frac{\partial \hat{\tau}_i}{\partial t_{ij}} + \frac{\partial r_{ij}}{\partial t_{ij}} \right) \right) \\ &+ V' \left( \tau_i \sum_{k \neq j} K_{ik}' \frac{\partial \rho}{\partial t_{ij}} + \tau_i K_{ij}' \frac{\partial r_{ij}}{\partial t_{ij}} + \frac{\partial \hat{\tau}_i}{\partial t_{ij}} \sum_{j=1}^{nm/\tilde{n}} K_{ij} + \tau_i \sum_{j=1}^{nm/\tilde{n}} K_{ij}' \frac{\partial r_{ij}}{\partial \tau_i} \frac{\partial \hat{\tau}_i}{\partial t_{ij}} \right) = 0. \end{aligned} \tag{14}$$

Invoking symmetry and factorizing, the local FOC (14) can be rewritten as: $(v'-1)\overline{k} + t_{ij} v' K_{ij}' \frac{\partial r_{ij}}{\partial t_{ij}} + V' \tau_i K_{ij}' \frac{\tilde{n}-1}{\tilde{n}} + \frac{\partial \hat{\tau}_i}{\partial t_{ij}} \left( \left( V' \frac{nm}{\tilde{n}} - 1 \right) \overline{k} + v' t_{ij} K_{ij}' \frac{\partial r_{ij}}{\partial \tau_i} + V' \tau_i \frac{nm}{\tilde{n}} K_{ij}' \frac{\partial r_{ij}}{\partial \tau_i} \right) = 0$. Since $\left( V' \frac{nm}{\tilde{n}} - 1 \right) \overline{k} + v' t_{ij} K_{ij}' \frac{\partial r_{ij}}{\partial \tau_i} + V' \tau_i \frac{nm}{\tilde{n}} K_{ij}' \frac{\partial r_{ij}}{\partial \tau_i} = 0$ from the regional FOC (1), the local FOC (14) boils down to $(v'-1)\overline{k} + t_{ij} v' K_{ij}' \frac{\partial r_{ij}}{\partial t_{ij}} + V' \tau_i K_{ij}' \frac{\tilde{n}-1}{\tilde{n}} = 0$, i.e., the local FOC in the Nash game. As a consequence, we get that $\tau_i^{SL} = \tau_i^N$, $t_{ij}^{SL} = t_{ij}^N$, and $\frac{\partial t_{ij}^{SL}}{\partial \tilde{n}} = \frac{\partial t_{ij}^N}{\partial \tilde{n}} > 0, \frac{\partial \tau_i^{SL}}{\partial \tilde{n}} = \frac{\partial \tau_i^N}{\partial \tilde{n}} < 0, \frac{\partial T_{ij}^{SL}}{\partial \tilde{n}} = \frac{\partial T_{ij}^N}{\partial \tilde{n}} < 0$.

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